1 $y = \sin(x) \xrightarrow{\text{spiegelen in de } y\text{-as}} f(x) = \sin(-x) \Rightarrow f(x) = \sin(-x)$ heeft dezelfde grafiek als $y = -\sin(x)$.

$y = \cos(x) \xrightarrow{\text{spiegelen in de } y\text{-as}} g(x) = \cos(-x) \Rightarrow g(x) = \cos(-x)$ heeft dezelfde grafiek als $y = \cos(x)$.

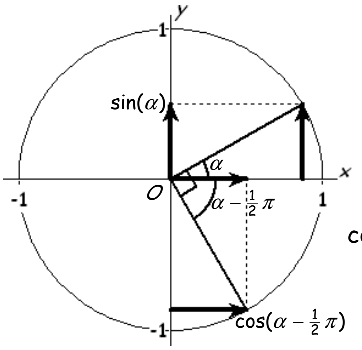
$y = \sin(x) \xrightarrow{\text{translatie } (-\frac{1}{2}\pi,\, 0)} h(x) = \sin(x + \frac{1}{2}\pi) \Rightarrow h(x) = \sin(x + \frac{1}{2}\pi)$ heeft dezelfde grafiek als $y = \cos(x)$.

$y = \cos(x) \xrightarrow{\text{translatie } (-\frac{1}{2}\pi,\, 0)} j(x) = \cos(x + \frac{1}{2}\pi) \Rightarrow j(x) = \cos(x + \frac{1}{2}\pi)$ heeft dezelfde grafiek als $y = -\sin(x)$.

$y = \sin(x) \xrightarrow{\text{translatie } (-\pi,\, 0)} k(x) = \sin(x + \pi) \Rightarrow k(x) = \sin(x + \pi)$ heeft dezelfde grafiek als $y = -\sin(x)$.

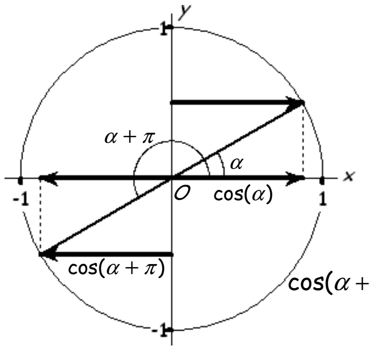
$y = \cos(x) \xrightarrow{\text{translatie } (-\pi,\, 0)} l(x) = \cos(x + \pi) \Rightarrow l(x) = \cos(x + \pi)$ heeft dezelfde grafiek als $y = -\cos(x)$.

2a



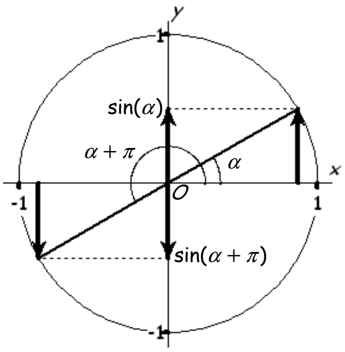
$\cos(\alpha - \frac{1}{2}\pi) = \sin(\alpha)$

2c



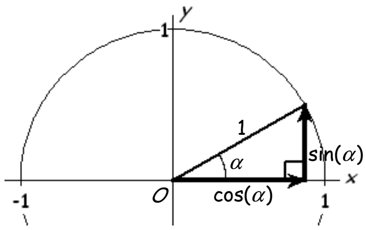
$\cos(\alpha + \pi) = -\cos(\alpha)$

2b



$\sin(\alpha + \pi) = -\sin(\alpha)$

2d



De stelling van Pythagoras geeft:
$\sin^2(\alpha) + \cos^2(\alpha) = 1$

3a $\sin(x + \frac{1}{6}\pi) = \cos(x + \frac{1}{6}\pi - \frac{1}{2}\pi) = \cos(x - \frac{1}{3}\pi)$.

3b $\cos(2x + \frac{1}{3}\pi) = \sin(2x + \frac{1}{3}\pi + \frac{1}{2}\pi) = \sin(2x + \frac{5}{6}\pi)$.

3c $-\sin(3x - \frac{2}{3}\pi) = \sin(3x - \frac{2}{3}\pi + \pi) = \sin(3x + \frac{1}{3}\pi) = \cos(3x + \frac{1}{3}\pi - \frac{1}{2}\pi) = \cos(3x - \frac{1}{6}\pi)$.

3d $-\cos(4x + 1\frac{1}{6}\pi) = \cos(4x + 1\frac{1}{6}\pi + \pi) = \cos(4x + 2\frac{1}{6}\pi) = \sin(4x + 2\frac{1}{6}\pi + \frac{1}{2}\pi) = \sin(4x + 2\frac{2}{3}\pi) = \sin(4x + \frac{2}{3}\pi)$.

4a $(\sin(x) - \cos(x))^2 = \sin^2(x) - 2\sin(x)\cos(x) + \cos^2(x) = \sin^2(x) + \cos^2(x) - 2\sin(x)\cos(x) = 1 - 2\sin(x)\cos(x)$

4b $\frac{2\sin^2(x) + \cos^2(x)}{\cos^2(x)} = \frac{2\sin^2(x)}{\cos^2(x)} + \frac{\cos^2(x)}{\cos^2(x)} = 2 \cdot \left(\frac{\sin(x)}{\cos(x)}\right)^2 + 1 = 2\tan^2(x) + 1.$

4c $\left(1 + \tan^2(3x)\right) \cdot \cos^2(3x) = \left(1 + \frac{\sin^2(3x)}{\cos^2(3x)}\right) \cdot \cos^2(3x) = \cos^2(3x) + \sin^2(3x) = 1.$

5a $\sin^2(x) + 4\cos(x) = 1 - \cos^2(x) + 4\cos(x).$

5b $2\cos^2(x) + \sin(x) - 2 = 2 \cdot \left(1 - \sin^2(x)\right) + \sin(x) - 2 = 2 - 2\sin^2(x) + \sin(x) - 2 = -2\sin^2(x) + \sin(x).$

5c $2\sin^2(x) + \cos^2(x) + \cos(x) = 2 \cdot \left(1 - \cos^2(x)\right) + \cos^2(x) + \cos(x)$

$= 2 - 2\cos^2(x) + \cos^2(x) + \cos(x) = 2 - \cos^2(x) + \cos(x).$

6 $\sin(2x - \frac{1}{3}\pi) = -\cos(x + \frac{1}{3}\pi)$

$\cos(2x - \frac{1}{3}\pi - \frac{1}{2}\pi) = \cos(x + \frac{1}{3}\pi + \pi)$

$\cos(2x - \frac{5}{6}\pi) = \cos(x + 1\frac{1}{3}\pi)$

hiernaast gaat het verder

$2x - \frac{5}{6}\pi = x + 1\frac{1}{3}\pi + k \cdot 2\pi \quad \vee \quad 2x - \frac{5}{6}\pi = -x - 1\frac{1}{3}\pi + k \cdot 2\pi$

$x = 2\frac{1}{6}\pi + k \cdot 2\pi \quad \vee \quad 3x = -\frac{1}{2}\pi + k \cdot 2\pi$

$x = 2\frac{1}{6}\pi + k \cdot 2\pi \quad \vee \quad x = -\frac{1}{6}\pi + k \cdot \frac{2}{3}\pi$

x op $[0, 2\pi]$ geeft $x = \frac{1}{6}\pi \vee x = \frac{1}{2}\pi \vee x = 1\frac{1}{6}\pi \vee x = 1\frac{5}{6}\pi$.

7a
$\sin(x + \frac{1}{2}\pi) = \cos(2x)$
$\cos(x + \frac{1}{2}\pi - \frac{1}{2}\pi) = \cos(2x)$
$x = 2x + k \cdot 2\pi \quad \vee \quad x = -2x + k \cdot 2\pi$
$-x = k \cdot 2\pi \quad \vee \quad 3x = k \cdot 2\pi$
$x = k \cdot 2\pi \quad \vee \quad x = k \cdot \frac{2}{3}\pi$
x op $[0, 2\pi] \Rightarrow x = 0 \vee x = \frac{2}{3}\pi \vee x = 1\frac{1}{3}\pi \vee x = 2\pi.$

7d
$\cos(x - 1) = -\cos(2x + 1)$
$\cos(x - 1) = \cos(2x + 1 + \pi)$
$x - 1 = 2x + 1 + \pi + k \cdot 2\pi \ \vee \ x - 1 = -2x - 1 - \pi + k \cdot 2\pi$
$-x = 2 + \pi + k \cdot 2\pi \ \vee \ 3x = -\pi + k \cdot 2\pi$
$x = -2 - \pi + k \cdot 2\pi \ \vee \ x = -\frac{1}{3}\pi + k \cdot \frac{2}{3}\pi$
x op $[0, 2\pi] \Rightarrow x = -2 + \pi \vee x = \frac{1}{3}\pi \vee x = \pi \vee x = 1\frac{2}{3}\pi.$

7b
$\sin(3x) = -\cos(x)$
$\cos(3x - \frac{1}{2}\pi) = \cos(x + \pi)$
$3x - \frac{1}{2}\pi = x + \pi + k \cdot 2\pi \quad \vee \quad 3x - \frac{1}{2}\pi = -x - \pi + k \cdot 2\pi$
$2x = 1\frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad 4x = -\frac{1}{2}\pi + k \cdot 2\pi$
$x = \frac{3}{4}\pi + k \cdot \pi \quad \vee \quad x = -\frac{1}{8}\pi + k \cdot \frac{1}{2}\pi$
x op $[0, 2\pi] \Rightarrow x = \frac{3}{4}\pi \vee x = 1\frac{3}{4}\pi \vee x = \frac{3}{8}\pi \vee x = \frac{7}{8}\pi \vee x = 1\frac{3}{8}\pi \vee x = 1\frac{7}{8}\pi.$

7e
$\sin(2x + \pi) = 1 - 2\sin(2x)$
$-\sin(2x) = 1 - 2\sin(2x)$
$\sin(2x) = 1$
$2x = \frac{1}{2}\pi + k \cdot 2\pi$
$x = \frac{1}{4}\pi + k \cdot \pi$
x op $[0, 2\pi] \Rightarrow x = \frac{1}{4}\pi \ \vee \ x = 1\frac{1}{4}\pi.$

7c
$\sin^2(x) + \frac{1}{2}\cos(x) = 1$
$1 - \cos^2(x) + \frac{1}{2}\cos(x) = 1$
$-\cos^2(x) + \frac{1}{2}\cos(x) = 0$
$-\cos(x) \cdot \left(\cos(x) - \frac{1}{2}\right) = 0$
$\cos(x) = 0 \vee \cos(x) = \frac{1}{2}$
$x = \frac{1}{2}\pi + k \cdot \pi \vee x = \frac{1}{3}\pi + k \cdot 2\pi \vee x = -\frac{1}{3}\pi + k \cdot 2\pi$
x op $[0, 2\pi] \Rightarrow x = \frac{1}{2}\pi \vee x = 1\frac{1}{2}\pi \vee x = \frac{1}{3}\pi \vee x = 1\frac{2}{3}\pi.$

7f
$2\sin^2(x) + \cos^2(x) + \cos(x) = 0$
$2 \cdot \left(1 - \cos^2(x)\right) + \cos^2(x) + \cos(x) = 0$
$2 - 2\cos^2(x) + \cos^2(x) + \cos(x) = 0$
$-\cos^2(x) + \cos(x) + 2 = 0$
$\cos^2(x) - \cos(x) - 2 = 0$
$\left(\cos(x) - 2\right) \cdot \left(\cos(x) + 1\right) = 0$
$\cos(x) = 2$ (kan niet) $\vee \cos(x) = -1$
$x = -\pi + k \cdot 2\pi$
x op $[0, 2\pi] \Rightarrow x = \pi.$

8a
$\cos(2\pi t) = \sin(\frac{1}{2}\pi t)$
$\cos(2\pi t) = \cos(\frac{1}{2}\pi t - \frac{1}{2}\pi)$
$2\pi t = \frac{1}{2}\pi t - \frac{1}{2}\pi + k \cdot 2\pi \vee 2\pi t = -\frac{1}{2}\pi t + \frac{1}{2}\pi + k \cdot 2\pi$
$1\frac{1}{2}\pi t = -\frac{1}{2}\pi + k \cdot 2\pi \vee 2\frac{1}{2}\pi t = \frac{1}{2}\pi + k \cdot 2\pi$
$t = -\frac{1}{3} + k \cdot \frac{4}{3} \vee t = \frac{1}{5} + k \cdot \frac{4}{5}$
t op $[0, 3] \Rightarrow t = 1 \vee t = 2\frac{1}{3} \vee t = \frac{1}{5} \vee t = 1\frac{4}{5} \vee t = 2\frac{3}{5}.$

8b
$\sin(\frac{\pi t}{6}) = -\cos(\pi t)$
$\cos(\frac{\pi t}{6} - \frac{1}{2}\pi) = \cos(\pi t + \pi)$
$\frac{\pi t}{6} - \frac{1}{2}\pi = \pi t + \pi + k \cdot 2\pi \vee \frac{\pi t}{6} - \frac{1}{2}\pi = -\pi t - \pi + k \cdot 2\pi$
$-\frac{5\pi t}{6} = 1\frac{1}{2}\pi + k \cdot 2\pi \vee \frac{7\pi t}{6} = -\frac{1}{2}\pi + k \cdot 2\pi$
$t = -\frac{9}{5} + k \cdot \frac{12}{5} \vee t = -\frac{3}{7} + k \cdot \frac{12}{7}$
t op $[0, 3] \Rightarrow t = \frac{3}{5} \vee t = 3 \vee t = 1\frac{2}{7}.$

9a
$2\sin(x) = \sin(x)$
$\sin(x) = 0$
$x = k \cdot \pi$

9b
$\sin(2x) = \sin(x)$
$2x = x + k \cdot 2\pi \vee 2x = \pi - x + k \cdot 2\pi$
$x = k \cdot 2\pi \vee 3x = \pi + k \cdot 2\pi$
$x = k \cdot 2\pi \vee x = \frac{1}{3}\pi + k \cdot \frac{2}{3}\pi$

9c ---

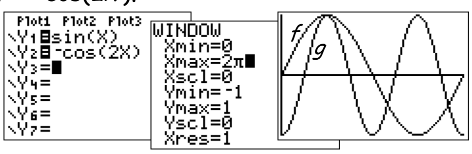
9d ---

9e
$\sin(2x) = \sin(x + \frac{1}{3}\pi)$
$2x = x + \frac{1}{3}\pi + k \cdot 2\pi \vee 2x = \pi - \left(x + \frac{1}{3}\pi\right) + k \cdot 2\pi$
$x = \frac{1}{3}\pi + k \cdot 2\pi \vee 2x = \pi - x - \frac{1}{3}\pi + k \cdot 2\pi$
$x = \frac{1}{3}\pi + k \cdot 2\pi \vee 3x = \frac{2}{3}\pi + k \cdot 2\pi$
$x = \frac{1}{3}\pi + k \cdot 2\pi \vee x = \frac{2}{9}\pi + k \cdot \frac{2}{3}\pi$

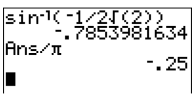
9f ---

10a $y = \cos(x) \xrightarrow{\text{verm. in de } y\text{-as, } \frac{1}{2}} y = \cos(2x) \xrightarrow{\text{verm. in de } x\text{-as, } -1} g(x) = -\cos(2x).$

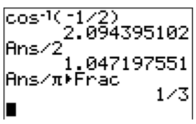
10b Zie de schets hiernaast.



10c
$f(x) = -\frac{1}{2}\sqrt{2} \Rightarrow \sin(x) = -\frac{1}{2}\sqrt{2}$
$x = -\frac{1}{4}\pi + k \cdot 2\pi \vee x = \pi - -\frac{1}{4}\pi + k \cdot 2\pi$
x op $[0, 2\pi] \Rightarrow x = \frac{7}{4}\pi = 1\frac{3}{4}\pi \vee x = \frac{5}{4}\pi = 1\frac{1}{4}\pi.$

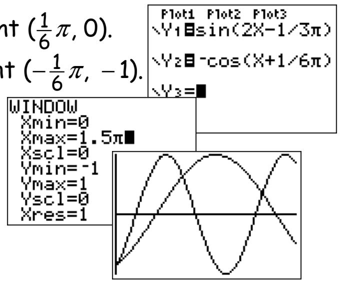


10d
$g(x) = \frac{1}{2} \Rightarrow -\cos(2x) = \frac{1}{2}$
$\cos(2x) = -\frac{1}{2}$
$2x = \pi - \frac{1}{3}\pi + k \cdot 2\pi \vee 2x = -\frac{2}{3}\pi + k \cdot 2\pi$
$x = \frac{1}{3}\pi + k \cdot \pi \vee x = -\frac{1}{3}\pi + k \cdot \pi$
x op $[0, 2\pi] \Rightarrow x = \frac{1}{3}\pi \vee x = 1\frac{1}{3}\pi \vee x = \frac{2}{3}\pi \vee x = 1\frac{2}{3}\pi.$



10e
$f(x) = g(x) \Rightarrow \sin(x) = -\cos(2x)$
$\cos(x - \frac{1}{2}\pi) = \cos(2x + \pi)$
$x - \frac{1}{2}\pi = 2x + \pi + k \cdot 2\pi \vee x - \frac{1}{2}\pi = -2x - \pi + k \cdot 2\pi$
$-x = 1\frac{1}{2}\pi + k \cdot 2\pi \vee 3x = -\frac{1}{2}\pi + k \cdot 2\pi$
$x = -1\frac{1}{2}\pi + k \cdot 2\pi \vee x = -\frac{1}{6}\pi + k \cdot \frac{2}{3}\pi$
x op $[0, 2\pi] \Rightarrow x = \frac{1}{2}\pi \vee x = 1\frac{1}{6}\pi \vee x = 1\frac{5}{6}\pi.$
$f(x) \le g(x)$ (zie de schets) $\Rightarrow x = \frac{1}{2}\pi \vee 1\frac{1}{6}\pi \le x \le 1\frac{5}{6}\pi.$

11a $f(x)=\sin(2x-\frac{1}{3}\pi)$ heeft evenwichtsstand 0; amplitude 1; periode $\frac{2\pi}{2}=\pi$ en beginpunt $(\frac{1}{6}\pi, 0)$.
$g(x)=-\cos(x+\frac{1}{6}\pi)$ heeft evenwichtsstand 0; amplitude 1; periode 2π en laagste punt $(-\frac{1}{6}\pi, -1)$.



11b Gebruik de plot hiernaast voor een schets van de grafieken.

11c $f(x)=\sin(2x-\frac{1}{3}\pi)=0$
$2x-\frac{1}{3}\pi=k\cdot\pi$
$2x=\frac{1}{3}\pi+k\cdot\pi$
$x=\frac{1}{6}\pi+k\cdot\frac{1}{2}\pi$
x op $[0, 1\frac{1}{2}\pi] \Rightarrow x=\frac{1}{6}\pi \vee x=\frac{2}{3}\pi \vee x=1\frac{1}{6}\pi$.
De nulpunten van f zijn $\frac{1}{6}\pi$, $\frac{2}{3}\pi$ en $1\frac{1}{6}\pi$.

$g(x)=-\cos(x+\frac{1}{6}\pi)=0$
$\cos(x+\frac{1}{6}\pi)=0$
$x+\frac{1}{6}\pi=\frac{1}{2}\pi+k\cdot\pi$
$x=\frac{1}{3}\pi+k\cdot\pi$
x op $[0, 1\frac{1}{2}\pi] \Rightarrow x=\frac{1}{3}\pi \vee x=1\frac{1}{3}\pi$.
De nulpunten van g zijn $\frac{1}{3}\pi$ en $x=1\frac{1}{3}\pi$.

11d $f(x)=\frac{1}{2} \Rightarrow \sin(2x-\frac{1}{3}\pi)=\frac{1}{2}$
$2x-\frac{1}{3}\pi=\frac{1}{6}\pi+k\cdot 2\pi \vee 2x-\frac{1}{3}\pi=\pi-\frac{1}{6}\pi+k\cdot 2\pi$
$2x=\frac{1}{2}\pi+k\cdot 2\pi \vee 2x=1\frac{1}{6}\pi+k\cdot 2\pi$
$x=\frac{1}{4}\pi+k\cdot\pi \vee x=\frac{7}{12}\pi+k\cdot\pi$
x op $[0, 1\frac{1}{2}\pi] \Rightarrow x=\frac{1}{4}\pi \vee x=1\frac{1}{4}\pi \vee x=\frac{7}{12}\pi$.
$f(x)>\frac{1}{2}$ (zie plot) $\Rightarrow \frac{1}{4}\pi<x<\frac{7}{12}\pi \vee 1\frac{1}{4}\pi<x\le 1\frac{1}{2}\pi$.

11e $f(x)=g(x) \Rightarrow \sin(2x-\frac{1}{3}\pi)=-\cos(x+\frac{1}{6}\pi)$
$\cos(2x-\frac{1}{3}\pi-\frac{1}{2}\pi)=\cos(x+\frac{1}{6}\pi+\pi)$
$\cos(2x-\frac{5}{6}\pi)=\cos(x+1\frac{1}{6}\pi)$
$2x-\frac{5}{6}\pi=x+1\frac{1}{6}\pi+k\cdot 2\pi \vee 2x-\frac{5}{6}\pi=-x-1\frac{1}{6}\pi+k\cdot 2\pi$
$x=2\pi+k\cdot 2\pi \vee 3x=-\frac{1}{3}\pi+k\cdot 2\pi$
$x=2\pi+k\cdot 2\pi \vee x=-\frac{1}{9}\pi+k\cdot\frac{2}{3}\pi$
x op $[0, 1\frac{1}{2}\pi] \Rightarrow x=0 \vee x=\frac{5}{9}\pi \vee x=1\frac{2}{9}\pi$.
$f(x)<g(x)$ (zie plot) $\Rightarrow \frac{5}{9}\pi<x<1\frac{2}{9}\pi$.

12a $AB=y_A-y_B=2\cdot y_A=2\sin(\alpha)$.

12b $AB^2=OA^2+OB^2-2\cdot OA\cdot OB\cdot\cos\angle AOB$
$(2\sin(\alpha))^2=1^2+1^2-2\cdot 1\cdot 1\cdot\cos(2\alpha)$
$4\sin^2(\alpha)=2-2\cos(2\alpha)$
$2\cos(2\alpha)=2-4\sin^2(\alpha)$
$\cos(2\alpha)=1-2\sin^2(\alpha)$.

13a $\cos(t-u)=\cos(t)\cdot\cos(u)+\sin(t)\cdot\sin(u)$
u vervangen door $-u$ geeft
$\cos(t-(-u))=\cos(t)\cdot\cos(-u)+\sin(t)\cdot\sin(-u)$
$\cos(t+u)=\cos(t)\cdot\cos(u)+\sin(t)\cdot-\sin(u)$
$\cos(t+u)=\cos(t)\cdot\cos(u)-\sin(t)\cdot\sin(u)$.

13b $\cos(t+u)=\cos(t)\cdot\cos(u)-\sin(t)\cdot\sin(u)$
u vervangen door $u-\frac{1}{2}\pi$ geeft
$\cos(t+u-\frac{1}{2}\pi)=\cos(t)\cdot\cos(u-\frac{1}{2}\pi)-\sin(t)\cdot\sin(u-\frac{1}{2}\pi)$
$\sin(t+u)=\cos(t)\cdot\sin(u)-\sin(t)\cdot-\cos(u)$
$\sin(t+u)=\cos(t)\cdot\sin(u)+\sin(t)\cdot\cos(u)$
$\sin(t+u)=\sin(t)\cdot\cos(u)+\cos(t)\cdot\sin(u)$.

13c $\sin(t+u)=\sin(t)\cdot\cos(u)+\cos(t)\cdot\sin(u)$
u vervangen door $-u$ geeft
$\sin(t+(-u))=\sin(t)\cdot\cos(-u)+\cos(t)\cdot\sin(-u)$
$\sin(t-u)=\sin(t)\cdot\cos(u)+\cos(t)\cdot-\sin(u)$
$\sin(t-u)=\sin(t)\cdot\cos(u)-\cos(t)\cdot\sin(u)$.

14a $\sin(t+u)=\sin(t)\cdot\cos(u)+\cos(t)\cdot\sin(u)$
t en u beide vervangen door A geeft
$\sin(A+A)=\sin(A)\cdot\cos(A)+\cos(A)\cdot\sin(A)$
$\sin(2A)=2\sin(A)\cdot\cos(A)$.
$\cos(t+u)=\cos(t)\cdot\cos(u)-\sin(t)\cdot\sin(u)$
t en u beide vervangen door A geeft
$\cos(A+A)=\cos(A)\cdot\cos(A)-\sin(A)\cdot\sin(A)$
$\cos(2A)=\cos^2(A)-\sin^2(A)$.

14b $\cos(2A)=\cos^2(A)-\sin^2(A)$ (zie 14a)
$\cos(2A)=\cos^2(A)-\left(1-\cos^2(A)\right)$
$\cos(2A)=\cos^2(A)-1+\cos^2(A)$
$\cos(2A)=2\cos^2(A)-1$.
$\cos(2A)=\cos^2(A)-\sin^2(A)$
$\cos(2A)=1-\sin^2(A)-\sin^2(A)$
$\cos(2A)=1-2\sin^2(A)$ (zie ook 12b).

15a $\cos(2A)=2\cos^2(A)-1$
$-2\cos^2(A)=-1-\cos(2A)$
$\cos^2(A)=\frac{1}{2}+\frac{1}{2}\cos(2A)$.

15b $\cos(2A)=1-2\sin^2(A)$
$2\sin^2(A)=1-\cos(2A)$
$\sin^2(A)=\frac{1}{2}-\frac{1}{2}\cos(2A)$.

16a $\sin(x)\cdot\cos(x)=\frac{1}{2}\sin(x-1)$
$\frac{1}{2}\cdot 2\cdot\sin(x)\cdot\cos(x)=\frac{1}{2}\sin(x-1)$
$\frac{1}{2}\sin(2x)=\frac{1}{2}\sin(x-1)$
$\sin(2x)=\sin(x-1)$
$2x=x-1+k\cdot 2\pi \vee 2x=\pi-x+1+k\cdot 2\pi$
$x=-1+k\cdot 2\pi \vee 3x=\pi+1+k\cdot 2\pi$
$x=-1+k\cdot 2\pi \vee x=\frac{1}{3}\pi+\frac{1}{3}+k\cdot\frac{2}{3}\pi$.

16c $\sin^2(\frac{1}{2}x)=\cos(x)+1\frac{1}{4}$
Gebruik: $\cos(2A)=1-2\sin^2(A) \Rightarrow \sin^2(A)=\frac{1}{2}-\frac{1}{2}\cos(2A)$
$\frac{1}{2}-\frac{1}{2}\cos(x)=\cos(x)+1\frac{1}{4}$
$-1\frac{1}{2}\cos(x)=\frac{3}{4}$
$\cos(x)=-\frac{1}{2}$
$x=\frac{2}{3}\pi+k\cdot 2\pi \vee x=-\frac{2}{3}\pi+k\cdot 2\pi$.

```
(3/4)/-1.5▸Frac
                -1/2
cos⁻¹(Ans)
         2.094395102
Ans/π▸Frac
                 2/3
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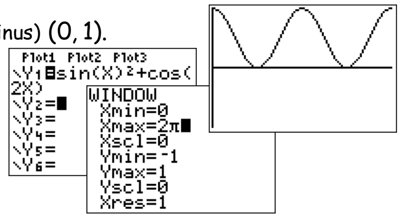
16b $\cos^2(2x) = \cos(4x) + \frac{1}{2}$
$\cos^2(2x) = 2\cos^2(2x) - 1 + \frac{1}{2}$
$-\cos^2(2x) = -\frac{1}{2}$
$\cos^2(2x) = \frac{1}{2}$
$\cos(2x) = \pm\sqrt{\frac{1}{2}} = \pm\sqrt{\frac{1}{2} \cdot \frac{2}{2}} = \pm\frac{1}{2}\sqrt{2}$
$2x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$
$x = \frac{1}{8}\pi + k \cdot \frac{1}{4}\pi.$

16d $(\sin(x) + \cos(x))^2 = 1\frac{1}{2}$
$\sin^2(x) + 2\sin(x)\cos(x) + \cos^2(x) = 1\frac{1}{2}$
$\sin^2(x) + \cos^2(x) + \sin(2x) = 1\frac{1}{2}$
$\sin(2x) = \frac{1}{2}$
$2x = \frac{1}{6}\pi + k \cdot 2\pi \vee 2x = \pi - \frac{1}{6}\pi + k \cdot 2\pi$
$x = \frac{1}{12}\pi + k \cdot \pi \vee x = \frac{5}{12}\pi + k \cdot \pi.$

17a Evenwichtsstand $\frac{1}{2}$; amplitude $\frac{1}{2}$; periode π en beginpunt (hoogste punt bij cosinus) $(0, 1)$.
Dus $y = \frac{1}{2} + \frac{1}{2}\cos(2x)$.



17b $\cos(2A) = 1 - 2\sin^2(A) \Rightarrow \sin^2(A) = \frac{1}{2} - \frac{1}{2}\cos(2A)$
$y = \sin^2(x) + \cos(2x) = \frac{1}{2} - \frac{1}{2}\cos(2x) + \cos(2x) = \frac{1}{2} + \frac{1}{2}\cos(2x).$

18 $\sin(3x) = \sin(2x + x)$
$= \sin(2x) \cdot \cos(x) + \cos(2x) \cdot \sin(x)$
$= 2\sin(x) \cdot \cos(x) \cdot \cos(x) + \left(1 - 2\sin^2(x)\right) \cdot \sin(x)$
$= 2\sin(x) \cdot \cos^2(x) + \sin(x) - 2\sin^3(x)$
$= 2\sin(x) \cdot \left(1 - \sin^2(x)\right) + \sin(x) - 2\sin^3(x)$
$= 2\sin(x) - 2\sin^3(x) + \sin(x) - 2\sin^3(x)$
$= 3\sin(x) - 4\sin^3(x).$

19b $\cos(3x) = \cos(2x + x)$
$= \cos(2x) \cdot \cos(x) - \sin(2x) \cdot \sin(x)$
$= \left(2\cos^2(x) - 1\right) \cdot \cos(x) - 2\sin(x)\cos(x) \cdot \sin(x)$
$= 2\cos^3(x) - \cos(x) - 2\sin^2(x)\cos(x)$
$= 2\cos^3(x) - \cos(x) - 2 \cdot \left(1 - \cos^2(x)\right) \cdot \cos(x)$
$= 2\cos^3(x) - \cos(x) - 2\cos(x) + 2\cos^3(x)$
$= 4\cos^3(x) - 3\cos(x).$

19a $\cos(2A) = 1 - 2\sin^2(A) \Rightarrow \sin^2(A) = \frac{1}{2} - \frac{1}{2}\cos(2A).$
$y = 1 - \cos(x) - \sin^2(\frac{1}{2}x) = 1 - \cos(x) - \left(\frac{1}{2} - \frac{1}{2}\cos(x)\right) = \frac{1}{2} - \frac{1}{2}\cos(x).$

19b staat hierboven uitgewerkt.

20a $-\cos(A) = \cos(A + \pi).$

20b $\sin(A) = \cos(A - \frac{1}{2}\pi).$

20c $\cos(2A) = 1 - 2\sin^2(A)$ of $\sin^2(A) = \frac{1}{2} - \frac{1}{2}\cos(2A).$

20d $\cos(2A) = 2\cos^2(A) - 1$ of $\cos^2(A) = \frac{1}{2} + \frac{1}{2}\cos(2A).$

20e $\cos(A) = \sin(A + \frac{1}{2}\pi).$

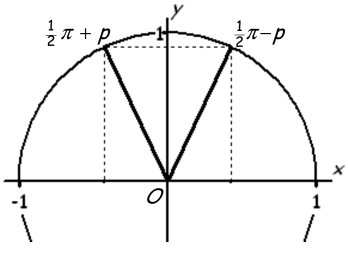
21 Voor B geldt: $x_B = -x_A$ en $y_B = y_A$. Voor C geldt: $x_C = -x_A$ en $y_C = -y_A$.

22a Voor elke p geldt: $f(-p) = -p\cos(-p) = -p \cdot \cos(p) = -f(p).$
$f(-p) = -f(p) \Rightarrow f(-p) + f(p) = 0 \Rightarrow f$ is (punt)symmetrisch in O.

22b Voor elke p geldt: $g(-p) = -p\sin(-p) = -p \cdot -\sin(p) = p\sin(p) = g(p) \Rightarrow g$ is (lijn)symmetrisch in de y-as.

23a Voor elke p geldt: $f(-p) = \cos^2(-p)\sin(-p) = \cos^2(p) \cdot -\sin(p) = -\cos^2(p)\sin(p) = -f(p).$
$f(-p) = -f(p) \Rightarrow f(-p) + f(p) = 0 \Rightarrow f$ is symmetrisch in O.

23b $f(\frac{1}{2}\pi - p) = \cos^2(\frac{1}{2}\pi - p)\sin(\frac{1}{2}\pi - p) = \cos^2(\frac{1}{2}\pi + p)\sin(\frac{1}{2}\pi + p) = f(\frac{1}{2}\pi + p).$
Dus f is symmetrisch in de lijn $x = \frac{1}{2}\pi$.
Gebruik: $\cos(\frac{1}{2}\pi - p) = -\cos(\frac{1}{2}\pi + p)$ (kwadr.)
$\cos^2(\frac{1}{2}\pi - p) = --\cos^2(\frac{1}{2}\pi + p) \Rightarrow \cos^2(\frac{1}{2}\pi - p) = \cos^2(\frac{1}{2}\pi + p).$



24a $f(-\frac{1}{4}\pi - p) = 2\sin(-\frac{1}{4}\pi - p) - 2\cos(-\frac{1}{4}\pi - p)$
$= 2\left(\sin(-\frac{1}{4}\pi)\cos(p) - \cos(-\frac{1}{4}\pi)\sin(p)\right) - 2\left(\cos(-\frac{1}{4}\pi)\cos(p) + \sin(-\frac{1}{4}\pi)\sin(p)\right)$
$= 2\left(-\frac{1}{2}\sqrt{2} \cdot \cos(p) - \frac{1}{2}\sqrt{2} \cdot \sin(p)\right) - 2\left(\frac{1}{2}\sqrt{2} \cdot \cos(p) + -\frac{1}{2}\sqrt{2} \cdot \sin(p)\right)$
$= -\sqrt{2} \cdot \cos(p) - \sqrt{2} \cdot \sin(p) - \sqrt{2} \cdot \cos(p) + \sqrt{2} \cdot \sin(p) = -2\sqrt{2} \cdot \cos(p).$

24b $f(-\frac{1}{4}\pi + p) = 2\sin(-\frac{1}{4}\pi + p) - 2\cos(-\frac{1}{4}\pi + p)$

$= 2\left(\sin(-\frac{1}{4}\pi)\cos(p) + \cos(-\frac{1}{4}\pi)\sin(p)\right) - 2\left(\cos(-\frac{1}{4}\pi)\cos(p) - \sin(-\frac{1}{4}\pi)\sin(p)\right)$

$= 2\left(-\frac{1}{2}\sqrt{2}\cdot\cos(p) + \frac{1}{2}\sqrt{2}\cdot\sin(p)\right) - 2\left(\frac{1}{2}\sqrt{2}\cdot\cos(p) - -\frac{1}{2}\sqrt{2}\cdot\sin(p)\right)$

$= -\sqrt{2}\cdot\cos(p) + \sqrt{2}\cdot\sin(p) - \sqrt{2}\cdot\cos(p) - \sqrt{2}\cdot\sin(p) = -2\sqrt{2}\cdot\cos(p).$

Voor elke p geldt: $f(-\frac{1}{4}\pi - p) = f(-\frac{1}{4}\pi + p) \Rightarrow f$ is symmetrisch in de lijn $x = -\frac{1}{4}\pi$.

25a $f(\frac{1}{4}\pi - p) = \cos(\frac{1}{4}\pi - p) + \sin(\frac{1}{4}\pi - p) + 1$

$= \cos(\frac{1}{4}\pi)\cos(p) + \sin(\frac{1}{4}\pi)\sin(p) + \sin(\frac{1}{4}\pi)\cos(p) - \cos(\frac{1}{4}\pi)\sin(p) + 1$

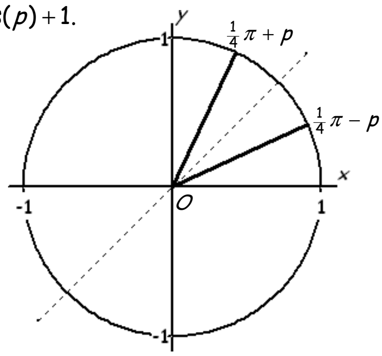
$= \frac{1}{2}\sqrt{2}\cdot\cos(p) + \frac{1}{2}\sqrt{2}\cdot\sin(p) + \frac{1}{2}\sqrt{2}\cdot\cos(p) - \frac{1}{2}\sqrt{2}\cdot\sin(p) + 1 = \sqrt{2}\cdot\cos(p) + 1.$

$f(\frac{1}{4}\pi + p) = \cos(\frac{1}{4}\pi + p) + \sin(\frac{1}{4}\pi + p) + 1$

$= \cos(\frac{1}{4}\pi)\cos(p) - \sin(\frac{1}{4}\pi)\sin(p) + \sin(\frac{1}{4}\pi)\cos(p) + \cos(\frac{1}{4}\pi)\sin(p) + 1$

$= \frac{1}{2}\sqrt{2}\cdot\cos(p) - \frac{1}{2}\sqrt{2}\cdot\sin(p) + \frac{1}{2}\sqrt{2}\cdot\cos(p) + \frac{1}{2}\sqrt{2}\cdot\sin(p) + 1 = \sqrt{2}\cdot\cos(p) + 1.$

Er geldt: $f(\frac{1}{4}\pi - p) = f(\frac{1}{4}\pi + p) \Rightarrow f$ is symmetrisch in de lijn $x = \frac{1}{4}\pi$.



Alternatieve uitwerking

$f(\frac{1}{4}\pi + p) = \cos(\frac{1}{4}\pi + p) + \sin(\frac{1}{4}\pi + p) + 1$ (gebruik de eenheidscirkel hiernaast)

$= \sin(\frac{1}{4}\pi - p) + \cos(\frac{1}{4}\pi - p) + 1$

$= \cos(\frac{1}{4}\pi - p) + \sin(\frac{1}{4}\pi - p) + 1 = f(\frac{1}{4}\pi - p).$

$f(\frac{1}{4}\pi - p) = f(\frac{1}{4}\pi + p) \Rightarrow f$ is symmetrisch in de lijn $x = \frac{1}{4}\pi$.

25b $f(\frac{3}{4}\pi - p) = \cos(\frac{3}{4}\pi - p) + \sin(\frac{3}{4}\pi - p) + 1$

$= \cos(\frac{3}{4}\pi)\cos(p) + \sin(\frac{3}{4}\pi)\sin(p) + \sin(\frac{3}{4}\pi)\cos(p) - \cos(\frac{3}{4}\pi)\sin(p) + 1$

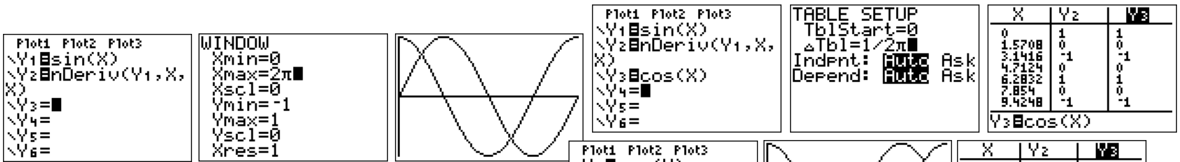
$= -\frac{1}{2}\sqrt{2}\cdot\cos(p) + \frac{1}{2}\sqrt{2}\cdot\sin(p) + \frac{1}{2}\sqrt{2}\cdot\cos(p) - -\frac{1}{2}\sqrt{2}\cdot\sin(p) + 1 = \sqrt{2}\cdot\sin(p) + 1.$

$f(\frac{3}{4}\pi + p) = \cos(\frac{3}{4}\pi + p) + \sin(\frac{3}{4}\pi + p) + 1$

$= \cos(\frac{3}{4}\pi)\cos(p) - \sin(\frac{3}{4}\pi)\sin(p) + \sin(\frac{3}{4}\pi)\cos(p) + \cos(\frac{3}{4}\pi)\sin(p) + 1$

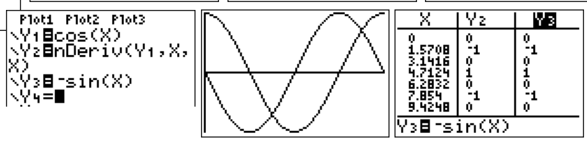
$= -\frac{1}{2}\sqrt{2}\cdot\cos(p) - \frac{1}{2}\sqrt{2}\cdot\sin(p) + \frac{1}{2}\sqrt{2}\cdot\cos(p) + -\frac{1}{2}\sqrt{2}\cdot\sin(p) + 1 = -\sqrt{2}\cdot\sin(p) + 1.$

$f(\frac{3}{4}\pi - p) + f(\frac{3}{4}\pi + p) = \sqrt{2}\cdot\sin(p) + 1 - \sqrt{2}\cdot\sin(p) + 1 = 2 \Rightarrow f$ is symmetrisch in het punt $(\frac{3}{4}\pi, 1)$.

26a 

26b $f(x) = \sin(x) \Rightarrow$ waarschijnlijk is $f'(x) = \cos(x)$.

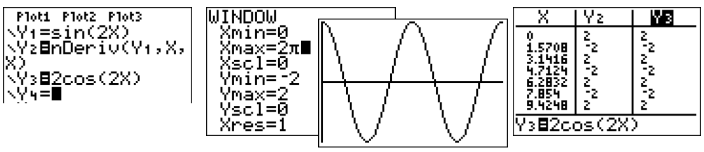
26c $f(x) = \cos(x) \Rightarrow$ waarschijnlijk is $f'(x) = -\sin(x)$.



27a Zie de plot van y_2 hiernaast.

27b $f(x) = \sin(\boxed{2x}) \Rightarrow f'(x) = 2\cos(2x)$.

27c $f(x) = \cos(\boxed{3x}) \Rightarrow f'(x) = -3\sin(3x)$.



28 $f(x) = \cos x = \sin(\boxed{x + \frac{1}{2}\pi}) \Rightarrow f'(x) = \cos(x + \frac{1}{2}\pi) = -\sin(x)$.

29 $f(x) = \sin(\boxed{ax + b}) \Rightarrow f'(x) = \cos(ax + b)\cdot a = a\cos(ax + b)$.

$g(x) = \cos(\boxed{ax + b}) \Rightarrow g'(x) = -\sin(ax + b)\cdot a = -a\sin(ax + b)$.

30a $f(x) = 3 + 4\sin(\boxed{2x - \frac{1}{3}\pi}) \Rightarrow f'(x) = 4\cos(2x - \frac{1}{3}\pi)\cdot 2 = 8\cos(2x - \frac{1}{3}\pi)$.

30b $g(x) = 10 + 16\cos(\boxed{\frac{1}{2}(\boxed{x - 1})}) \Rightarrow g'(x) = -16\sin(\frac{1}{2}(x - 1))\cdot\frac{1}{2}\cdot 1 = -8\sin(\frac{1}{2}(x - 1))$.

30c $h(x) = x\cos(x) \Rightarrow h'(x) = 1\cdot\cos(x) + x\cdot -\sin(x) = \cos(x) - x\sin(x)$.

30d $j(x) = x\cos(\boxed{2x}) \Rightarrow j'(x) = 1\cdot\cos(2x) + x\cdot -2\sin(2x) = \cos(2x) - 2x\sin(2x)$.

30e $k(x) = x^2 \cdot \sin(\boxed{3x}) \Rightarrow k'(x) = 2x \cdot \sin(3x) + x^2 \cdot 3\cos(3x) = 2x\sin(3x) + 3x^2\cos(3x).$

30f $l(x) = 2x \cdot \sin(\boxed{3x-1}) \Rightarrow l'(x) = 2 \cdot \sin(3x-1) + 2x \cdot 3\cos(3x-1) = 2\sin(3x-1) + 6x\cos(3x-1).$

31a $f(x) = 3\tan(\boxed{2x}) \Rightarrow f'(x) = 3 \cdot \frac{1}{\cos^2(2x)} \cdot 2 = \frac{6}{\cos^2(2x)}.$

31b $g(x) = \tan^2(x) = \left(\boxed{\tan(x)}\right)^2 \Rightarrow g'(x) = 2\tan(x) \cdot \frac{1}{\cos^2(x)} = 2 \cdot \frac{\sin(x)}{\cos(x)} \cdot \frac{1}{\cos^2(x)} = \frac{2\sin(x)}{\cos^3(x)}.$

31c $h(x) = \cos(x) \cdot \tan(x) = \cos(x) \cdot \frac{\sin(x)}{\cos(x)} = \sin(x) \Rightarrow h'(x) = \cos(x).$

32 I $f(x) = \sin^2(x) = \sin(x) \cdot \sin(x) \Rightarrow f'(x) = \cos(x) \cdot \sin(x) + \sin(x) \cdot \cos(x) = 2\sin(x)\cos(x).$

II $f(x) = \sin^2(x) = \left(\boxed{\sin(x)}\right)^2 \Rightarrow f'(x) = 2\sin(x) \cdot \cos(x).$

III $f(x) = \sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(\boxed{2x}) \Rightarrow f'(x) = -\frac{1}{2} \cdot -\sin(\boxed{2x}) \cdot 2 = \sin(2x).$

Mijn persoonlijke voorkeur gaat uit naar II omdat in dit geval $f(x)$ niet hoeft te worden herschreven.

33a $f(x) = \cos^2(x) = \left(\boxed{\cos(x)}\right)^2 \Rightarrow f'(x) = 2\cos(x) \cdot -\sin(x) = -2\sin(x)\cos(x).$

33b $g(x) = 2\sin^2(x) = 2\left(\boxed{\sin(x)}\right)^2 \Rightarrow g'(x) = 4\sin(x) \cdot \cos(x).$

33c $h(x) = 1 + 2\cos^2(x) = 1 + 2\left(\boxed{\cos(x)}\right)^2 \Rightarrow h'(x) = 4\cos(x) \cdot -\sin(x) = -4\sin(x)\cos(x).$

33d $j(x) = x + 3\sin^2(x) = x + 3\left(\boxed{\sin(x)}\right)^2 \Rightarrow j'(x) = 1 + 6\sin(x) \cdot \cos(x).$

34a $f(x) = \sin^3(x) = \left(\boxed{\sin(x)}\right)^3 \Rightarrow f'(x) = 3\sin^2(x) \cdot \cos(x).$

34b $g(x) = x \cdot \sin^2(x) = x \cdot \left(\boxed{\sin(x)}\right)^2 \Rightarrow g'(x) = 1 \cdot \sin^2(x) + x \cdot 2\sin(x) \cdot \cos(x) = \sin^2(x) + 2x\sin(x)\cos(x).$

34c $h(x) = \cos^2(2x) = \left(\boxed{\cos(\boxed{2x})}\right)^2 \Rightarrow h'(x) = 2\cos(2x) \cdot -\sin(2x) \cdot 2 = -4\sin(2x)\cos(2x).$

34d $j(x) = \cos^2(x^2) = \left(\boxed{\cos(\boxed{x^2})}\right)^2 \Rightarrow j'(x) = 2\cos(x^2) \cdot -\sin(x^2) \cdot 2x = -4x\sin(x^2)\cos(x^2).$

35a $f(x) = \sin^3(x) + \sin(x) = \left(\boxed{\sin(x)}\right)^3 + \sin(x) \Rightarrow f'(x) = 3\sin^2(x) \cdot \cos(x) + \cos(x)$

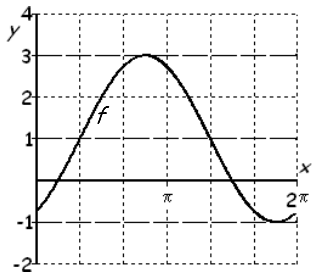
$= 3\cos(x) \cdot \left(1 - \cos^2(x)\right) + \cos(x) = 4\cos(x) - 3\cos^3(x).$

35b $g(x) = \sin^2(x) \cdot \cos(x) = \left(\boxed{\sin(x)}\right)^2 \cdot \cos(x) \Rightarrow$

$g'(x) = 2\sin(x) \cdot \cos(x) \cdot \cos(x) + \sin^2(x) \cdot -\sin(x) = 2\sin(x) \cdot \cos^2(x) - \sin^3(x)$

$= 2\sin(x) \cdot \left(1 - \sin^2(x)\right) - \sin^3(x) = 2\sin(x) - 2\sin^3(x) - \sin^3(x) = 2\sin(x) - 3\sin^3(x).$

35c $h(x) = \frac{\tan(x)}{\sin(x)} = \frac{\sin(x)}{\cos(x)} \cdot \frac{1}{\sin(x)} = \frac{1}{\cos(x)} = \left(\boxed{\cos(x)}\right)^{-1} \Rightarrow h'(x) = -1 \cdot (\cos(x))^{-2} \cdot -\sin(x) = \frac{\sin(x)}{\cos^2(x)}.$



36a $f(x) = 1 + 2\sin(x - \frac{1}{3}\pi)$ heeft evenwichtsstand 1;
amplitude 2; periode 2π en beginpunt $(\frac{1}{3}\pi, 1)$.

36b Horizontale raaklijnen in de toppen bij $x = \frac{1}{3}\pi + \frac{1}{4} \cdot 2\pi = \frac{5}{6}\pi$ en $x = \frac{5}{6}\pi + \frac{1}{2} \cdot 2\pi = 1\frac{5}{6}\pi.$

37a $f(x) = -2 + 2\sin(3x - \frac{1}{2}\pi) = -2 + 2\sin(3(x - \frac{1}{6}\pi))$ heeft
evenwichtsstand -2; amplitude 2; periode $\frac{2\pi}{3} = \frac{2}{3}\pi$ en beginpunt $(\frac{1}{6}\pi,\ -2)$.
Hoogste punten zijn $(\frac{1}{6}\pi + \frac{1}{4} \cdot \frac{2}{3}\pi + k \cdot \frac{2}{3}\pi,\ -2 + 2) = (\frac{1}{3}\pi + k \cdot \frac{2}{3}\pi,\ 0).$
Laagste punten zijn $(\frac{1}{6}\pi + \frac{3}{4} \cdot \frac{2}{3}\pi + k \cdot \frac{2}{3}\pi,\ -2 - 2) = (\frac{2}{3}\pi + k \cdot \frac{2}{3}\pi,\ -4).$
De toppen zijn $(\frac{1}{3}\pi + k \cdot \frac{2}{3}\pi, 0)$ en $(k \cdot \frac{2}{3}\pi,\ -4).$

37b $g(x) = 2 + \cos(\frac{1}{3}x + \frac{1}{2}\pi) = 2 + \cos(\frac{1}{3}(x + \frac{3}{2}\pi))$ heeft
evenwichtsstand 2; amplitude 1; periode $\frac{2\pi}{\frac{1}{3}} = 6\pi$ en beginpunt (hoogste punt) $(-1\frac{1}{2}\pi, 2+1) = (-1\frac{1}{2}\pi, 3)$.
Hoogste punten zijn $(-1\frac{1}{2}\pi + k \cdot 6\pi, 2+1) = (-1\frac{1}{2}\pi + k \cdot 6\pi, 3)$.
Laagste punten zijn $(-1\frac{1}{2}\pi + \frac{1}{2} \cdot 6\pi + k \cdot 6\pi, 2-1) = (1\frac{1}{2}\pi + k \cdot 6\pi, 1)$.

37c $h(x) = 1 - 3\sin(x + \frac{1}{6}\pi)$ heeft evenwichtsstand 1; amplitude 3; periode $\frac{2\pi}{1} = 2\pi$ en beginpunt $(-\frac{1}{6}\pi + \pi, 1) = (\frac{5}{6}\pi, 1)$.
Hoogste punten zijn $(\frac{5}{6}\pi + \frac{1}{4} \cdot 2\pi + k \cdot 2\pi, 1+3) = (1\frac{1}{3}\pi + k \cdot 2\pi, 4)$.
Laagste punten zijn $(\frac{5}{6}\pi + \frac{3}{4} \cdot 2\pi + k \cdot 2\pi, 1-3) = (\frac{1}{3}\pi + k \cdot 2\pi, -2)$.

beginpunt bij een sinus-grafiekis een punt waar de grafiek STIJGEND door de evenwichtsstand gaat

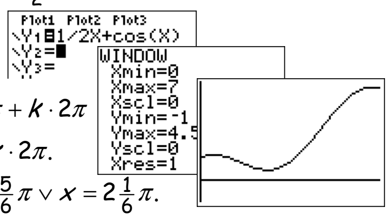
37d $j(x) = -2 - \cos(2x)$ heeft evenwichtsstand -2; ampl. 1; periode $\frac{2\pi}{2} = \pi$ en beginpunt $(0 + \frac{1}{2}\pi, -2+1) = (\frac{1}{2}\pi, -1)$.
Hoogste punten zijn $(\frac{1}{2}\pi + k \cdot \pi, -2+1) = (\frac{1}{2}\pi + k \cdot \pi, -1)$.
Laagste punten zijn $(\frac{1}{2}\pi + \frac{1}{2} \cdot \pi + k \cdot \pi, -2-1) = (k \cdot \pi, -3)$.

beginpunt van een cosinus-grafiek is een hoogste punt

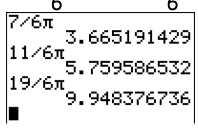
38a $f(x) = \cos(\boxed{2x}) - 2\sin(x) + 2 \Rightarrow f'(x) = -2\sin(2x) - 2\cos(x)$.
$f'(x) = 0 \Rightarrow -2\sin(2x) - 2\cos(x) = 0$
$\sin(2x) = -\cos(x)$
$\cos(2x - \frac{1}{2}\pi) = \cos(x + \pi)$
$2x - \frac{1}{2}\pi = x + \pi + k \cdot 2\pi \vee 2x - \frac{1}{2}\pi = -x - \pi + k \cdot 2\pi$
$x = 1\frac{1}{2}\pi + k \cdot 2\pi \vee 3x = -\frac{1}{2}\pi + k \cdot 2\pi$
$x = 1\frac{1}{2}\pi + k \cdot 2\pi \vee x = -\frac{1}{6}\pi + k \cdot \frac{2}{3}\pi$.
Dus $x_A = \frac{1}{2}\pi$; $x_B = 1\frac{1}{6}\pi$; $x_C = 1\frac{1}{2}\pi$; en $x_D = 1\frac{5}{6}\pi$.
$y_A = f(\frac{1}{2}\pi) = \cos(\pi) - 2\sin(\frac{1}{2}\pi) + 2 = -1 - 2 + 2 = -1 \Rightarrow A(\frac{1}{2}\pi, -1)$;
$y_B = f(1\frac{1}{6}\pi) = \cos(2\frac{1}{3}\pi) - 2\sin(1\frac{1}{6}\pi) + 2 = \frac{1}{2} - 2 \cdot -\frac{1}{2} + 2 = \frac{1}{2} + 1 + 2 = 3\frac{1}{2} \Rightarrow B(1\frac{1}{6}\pi, 3\frac{1}{2})$;
$y_C = f(1\frac{1}{2}\pi) = \cos(3\pi) - 2\sin(1\frac{1}{2}\pi) + 2 = -1 - 2 \cdot -1 + 2 = -1 + 2 + 2 = 3 \Rightarrow C(1\frac{1}{2}\pi, 3)$ en
$y_D = f(1\frac{5}{6}\pi) = \cos(3\frac{2}{3}\pi) - 2\sin(1\frac{5}{6}\pi) + 2 = \frac{1}{2} - 2 \cdot -\frac{1}{2} + 2 = \frac{1}{2} + 1 + 2 = 3\frac{1}{2} \Rightarrow D(1\frac{5}{6}\pi, 3\frac{1}{2})$.

38b $f(0) = f(2\pi) = \cos(0) - 2\sin(0) + 2 = 1 - 0 + 2 = 3$
Dus $f(x) = p$ heeft vier oplossingen (zie ook figuur 11.14) voor $3 \le p < 3\frac{1}{2}$.

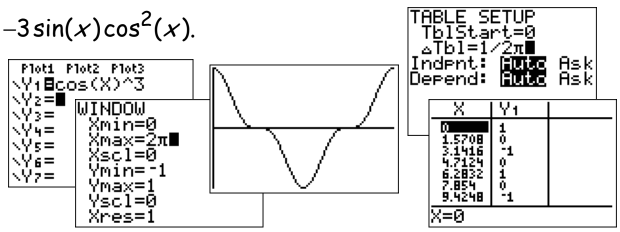
39a $f(x) = \frac{1}{2}x + \cos(x) \Rightarrow f'(x) = \frac{1}{2} - \sin(x)$.
$f'(x) = 0 \Rightarrow \frac{1}{2} - \sin(x) = 0$
$-\sin(x) = -\frac{1}{2} \Rightarrow \sin(x) = \frac{1}{2}$
$x = \frac{1}{6}\pi + k \cdot 2\pi \vee x = \pi - \frac{1}{6}\pi + k \cdot 2\pi$
$x = \frac{1}{6}\pi + k \cdot 2\pi \vee x = \frac{5}{6}\pi + k \cdot 2\pi$.
x op $[0, 7] \Rightarrow x = \frac{1}{6}\pi \vee x = \frac{5}{6}\pi \vee x = 2\frac{1}{6}\pi$.



39b $f'(x) = 1 \Rightarrow \frac{1}{2} - \sin(x) = 1$
$-\sin(x) = \frac{1}{2} \Rightarrow \sin(x) = -\frac{1}{2}$
$x = 1\frac{1}{6}\pi + k \cdot 2\pi \vee x = \pi - 1\frac{1}{6}\pi + k \cdot 2\pi$
x op $[0, 7] \Rightarrow x = 1\frac{1}{6}\pi \vee x = 1\frac{5}{6}\pi$.



40 $f(x) = \cos^3(x) = (\boxed{\cos(x)})^3 \Rightarrow f'(x) = 3\cos^2(x) \cdot -\sin(x) = -3\sin(x)\cos^2(x)$.
$f'(x) = 0 \Rightarrow -3\sin(x)\cos^2(x) = 0$
$\sin(x) = 0 \vee \cos(x) = 0$
$x = k \cdot \pi \vee x = \frac{1}{2}\pi + k \cdot \pi$
x op $[0, 2\pi] \Rightarrow x = 0 \vee x = \frac{1}{2}\pi \vee x = \pi \vee x = 1\frac{1}{2}\pi \vee x = 2\pi$.
De punten zijn $(0, 1)$; $(\frac{1}{2}\pi, 0)$; $(\pi, -1)$; $(1\frac{1}{2}\pi, 0)$ en $(2\pi, 1)$.

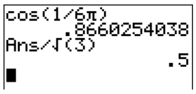


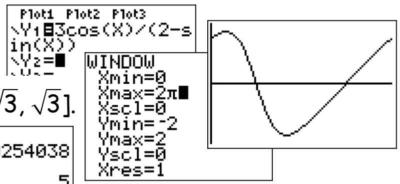
41a $f(x) = \frac{3\cos(x)}{2 - \sin(x)} \Rightarrow f'(x) = \frac{(2 - \sin(x)) \cdot -3\sin(x) - 3\cos(x) \cdot -\cos(x)}{(2 - \sin(x))^2} = \frac{-6\sin(x) + 3\sin^2(x) + 3\cos^2(x)}{(2 - \sin(x))^2} = \frac{-6\sin(x) + 3}{(2 - \sin(x))^2}$.
$f(x) = 0 \Rightarrow \frac{3\cos(x)}{2 - \sin(x)} = 0$ (teller = 0) $\Rightarrow \cos(x) = 0 \Rightarrow x = \frac{1}{2}\pi + k \cdot \pi$. Nu x op $[0, 2\pi] \Rightarrow x = \frac{1}{2}\pi \vee x = 1\frac{1}{2}\pi$.
$x = \frac{1}{2}\pi$ (en $y = 0$) $\Rightarrow S_1(\frac{1}{2}\pi, 0)$
$\text{rc} = f'(\frac{1}{2}\pi) = \frac{-6\sin(\frac{1}{2}\pi) + 3}{(2 - \sin(\frac{1}{2}\pi))^2} = \frac{-6 \cdot 1 + 3}{(2-1)^2} = \frac{-3}{1}$ $\Rightarrow$ $y = -3x + b$ door $S_1(\frac{1}{2}\pi, 0)$ $\Rightarrow 0 = -3 \cdot \frac{1}{2}\pi + b \Rightarrow b = 1\frac{1}{2}\pi$, dus k: $y = -3x + 1\frac{1}{2}\pi$.

$$\left.\begin{array}{l} x = 1\tfrac{1}{2}\pi \text{ (en } y=0) \Rightarrow S_2(1\tfrac{1}{2}\pi, 0) \\ \text{rc} = f'(1\tfrac{1}{2}\pi) = \dfrac{-6\sin(1\frac{1}{2}\pi)+3}{\left(2-\sin(1\frac{1}{2}\pi)\right)^2} = \dfrac{-6\cdot -1+3}{(2--1)^2} = \dfrac{9}{9} \end{array}\right\} \Rightarrow \left.\begin{array}{l} y = x + b \\ \text{door } S_2(1\tfrac{1}{2}\pi, 0) \end{array}\right\} \Rightarrow 0 = 1\tfrac{1}{2}\pi + b \Rightarrow b = -1\tfrac{1}{2}\pi, \text{ dus } l\text{: } y = x - 1\tfrac{1}{2}\pi.$$

41b $f'(x) = \dfrac{-6\sin(x)+3}{(2-\sin(x))^2} = 0$ (teller = 0) $\Rightarrow -6\sin(x)+3 = 0 \Rightarrow -6\sin(x) = -3 \Rightarrow \sin(x) = \frac{1}{2} \Rightarrow x = \frac{1}{6}\pi \vee x = \frac{5}{6}\pi$ (op $[0, 2\pi]$).

$$\left.\begin{array}{l} \text{randmaximum (zie plot) is } f(2\pi) = \dfrac{3\cos(2\pi)}{2-\sin(2\pi)} = \dfrac{3\cdot 1}{2-0} = 1\tfrac{1}{2} \\ \text{maximum (zie plot) is } f(\tfrac{1}{6}\pi) = \dfrac{3\cos(\frac{1}{6}\pi)}{2-\sin(\frac{1}{6}\pi)} = \dfrac{3\cdot\frac{1}{2}\sqrt{3}}{2-\frac{1}{2}} = \dfrac{1\frac{1}{2}\sqrt{3}}{1\frac{1}{2}} = \sqrt{3} > 1\tfrac{1}{2} \\ \text{minimum (zie plot) is } f(\tfrac{5}{6}\pi) = \dfrac{3\cos(\frac{5}{6}\pi)}{2-\sin(\frac{5}{6}\pi)} = \dfrac{3\cdot -\frac{1}{2}\sqrt{3}}{2-\frac{1}{2}} = \dfrac{-1\frac{1}{2}\sqrt{3}}{1\frac{1}{2}} = -\sqrt{3}. \end{array}\right\} \Rightarrow B_f = [-\sqrt{3}, \sqrt{3}].$$





42a $F(x) = -\frac{1}{3}\cos(\boxed{3x}) \Rightarrow F'(x) = -\frac{1}{3}\cdot -\sin(3x)\cdot 3 = \sin(3x) = f(x).$

42b $G(x) = \frac{1}{5}\sin(\boxed{5x}) \Rightarrow G'(x) = \frac{1}{5}\cdot\cos(5x)\cdot 5 = \cos(5x) = g(x).$

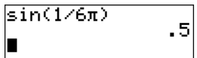
43a $f(x) = 4\sin(\boxed{\tfrac{1}{3}x}) \Rightarrow F(x) = 4\cdot\frac{1}{\frac{1}{3}}\cdot -\cos(\frac{1}{3}x) + c = -12\cos(\frac{1}{3}x) + c.$

43b $g(x) = x^2 - 5\cos(\boxed{2x}) \Rightarrow G(x) = \frac{1}{3}x^3 - 5\cdot\frac{1}{2}\cdot\sin(2x) + c = \frac{1}{3}x^3 - \frac{5}{2}\sin(2x) + c.$

43c $h(x) = \sin(\boxed{2x + \tfrac{1}{3}\pi}) \Rightarrow H(x) = \frac{1}{2}\cdot -\cos(2x + \frac{1}{3}\pi) + c = -\frac{1}{2}\cos(2x + \frac{1}{3}\pi) + c.$

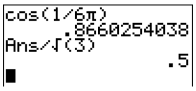
43d $j(x) = 3\cos(\boxed{\tfrac{1}{2}x - \tfrac{1}{6}\pi}) \Rightarrow J(x) = 3\cdot\frac{1}{\frac{1}{2}}\cdot\sin(\frac{1}{2}x - \frac{1}{6}\pi) + c = 6\sin(\frac{1}{2}x - \frac{1}{6}\pi) + c.$

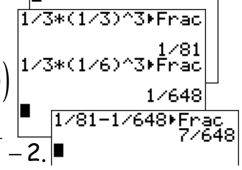
44a $\displaystyle\int_0^{\frac{1}{3}\pi}\left(2x + \cos(\boxed{\tfrac{1}{2}x})\right)dx = \left[x^2 + 2\sin(\tfrac{1}{2}x)\right]_0^{\frac{1}{3}\pi} = (\tfrac{1}{3}\pi)^2 + 2\sin(\tfrac{1}{6}\pi) - \left(0^2 + 2\cdot 0\right) = \tfrac{1}{9}\pi^2 + 2\cdot\tfrac{1}{2} = \tfrac{1}{9}\pi^2 + 1.$



44b $\displaystyle\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi}\left(x^2 - 2\sin(\boxed{x - \tfrac{1}{6}\pi})\right)dx = \left[\tfrac{1}{3}x^3 + 2\cos(x - \tfrac{1}{6}\pi)\right]_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} = \tfrac{1}{3}\cdot(\tfrac{1}{3}\pi)^3 + 2\cos(\tfrac{1}{6}\pi) - \left(\tfrac{1}{3}\cdot(\tfrac{1}{6}\pi)^3 + 2\cos(0)\right)$

$= \frac{1}{81}\pi^3 + 2\cdot\frac{1}{2}\sqrt{3} - \left(\frac{1}{648}\pi^3 + 2\cdot 1\right) = \frac{7}{648}\pi^3 + \sqrt{3} - 2.$

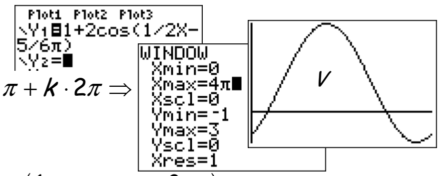




45 $f(x) = 1 + 2\cos(\frac{1}{2}x - \frac{5}{6}\pi) = 0 \Rightarrow 2\cos(\frac{1}{2}x - \frac{5}{6}\pi) = -1 \Rightarrow \cos(\frac{1}{2}x - \frac{5}{6}\pi) = -\frac{1}{2} \Rightarrow$
$\frac{1}{2}x - \frac{5}{6}\pi = \frac{2}{3}\pi + k\cdot 2\pi \vee \frac{1}{2}x - \frac{5}{6}\pi = -\frac{2}{3}\pi + k\cdot 2\pi \Rightarrow \frac{1}{2}x = \frac{9}{6}\pi + k\cdot 2\pi \vee \frac{1}{2}x = \frac{1}{6}\pi + k\cdot 2\pi \Rightarrow$
$x = 3\pi + k\cdot 4\pi \vee x = \frac{1}{3}\pi + k\cdot 4\pi$. Er geldt: x op $[0, 4\pi] \Rightarrow x = 3\pi \vee x = \frac{1}{3}\pi.$

$\displaystyle O(V) = \int_{\frac{1}{3}\pi}^{3\pi}\left(1 + 2\cos(\boxed{\tfrac{1}{2}x - \tfrac{5}{6}\pi})\right)dx = \left[x + 4\sin(\tfrac{1}{2}x - \tfrac{5}{6}\pi)\right]_{\frac{1}{3}\pi}^{3\pi} = 3\pi + 4\sin(\tfrac{2}{3}\pi) - \left(\tfrac{1}{3}\pi + 4\sin(-\tfrac{2}{3}\pi)\right)$

$= 3\pi + 4\cdot\frac{1}{2}\sqrt{3}) - \left(\frac{1}{3}\pi + 4\cdot -\frac{1}{2}\sqrt{3}\right) = 2\frac{2}{3}\pi + 4\sqrt{3}.$



46a $g(x) = \frac{1}{3}\sin^3(x) = \frac{1}{3}\left(\boxed{\sin(x)}\right)^3 \Rightarrow g'(x) = \frac{1}{3}\cdot 3\sin^2(x)\cdot\cos(x) \neq f(x)$. Dus $g(x) = \frac{1}{3}\sin^3(x)$ is geen primitieve van f.

46bc $\cos(2A) = 1 - 2\sin^2(A) \Rightarrow 2\sin^2(A) = 1 - \cos(2A) \Rightarrow \sin^2(A) = \frac{1}{2} - \frac{1}{2}\cos(2A).$
$f(x) = \sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(\boxed{2x}) \Rightarrow F(x) = \frac{1}{2}x - \frac{1}{2}\cdot\frac{1}{2}\cdot\sin(2x) + c = \frac{1}{2}x - \frac{1}{4}\sin(2x) + c.$

47a $\cos(2A) = 2\cos^2(A) - 1 \Rightarrow \cos(2A) + 1 = 2\cos^2(A) \Rightarrow \frac{1}{2}\cos(2A) + \frac{1}{2} = \cos^2(A).$
$f(x) = \cos^2(x) = \frac{1}{2}\cos(\boxed{2x}) + \frac{1}{2} \Rightarrow F(x) = \frac{1}{2}\cdot\frac{1}{2}\cdot\sin(2x) + \frac{1}{2}x + c = \frac{1}{4}\sin(2x) + \frac{1}{2}x + c.$

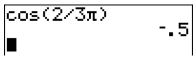
47b $\cos(2A) = 1 - 2\sin^2(A) \Rightarrow 2\sin^2(A) = 1 - \cos(2A) \Rightarrow \sin^2(A) = \frac{1}{2} - \frac{1}{2}\cos(2A).$
$g(x) = \sin^2(3x) = \frac{1}{2} - \frac{1}{2}\cos(\boxed{6x}) \Rightarrow G(x) = \frac{1}{2}x - \frac{1}{2}\cdot\frac{1}{6}\cdot\sin(6x) + c = \frac{1}{2}x - \frac{1}{12}\sin(6x) + c.$

47c $\sin(2A) = 2\sin(A)\cos(A) \Rightarrow \frac{1}{2}\sin(2A) = \sin(A)\cos(A).$
$h(x) = \sin(\frac{1}{2}x)\cos(\frac{1}{2}x) = \frac{1}{2}\sin(x) \Rightarrow H(x) = \frac{1}{2}\cdot -\cos(x) + c = -\frac{1}{2}\cos(x) + c.$

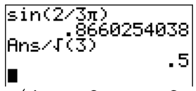
48a $f(x)=\tan^2(x)=\left(1+\tan^2(x)\right)-1 \Rightarrow F(x)=\tan(x)-x+c.$

48b $g(x)=x+\tan^2(x)=x+\left(1+\tan^2(x)\right)-1 \Rightarrow G(x)=\frac{1}{2}x^2+\tan(x)-x+c.$

49a $\sin(2A)=2\sin(A)\cos(A) \Rightarrow \frac{1}{2}\sin(2A)=\sin(A)\cos(A).$

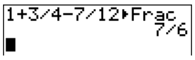


$$\int_0^{\frac{1}{6}\pi}\sin(2x)\cos(2x)\,dx=\int_0^{\frac{1}{6}\pi}\tfrac{1}{2}\sin(\boxed{4x})\,dx=\left[-\tfrac{1}{8}\cos(4x)\right]_0^{\frac{1}{6}\pi}=-\tfrac{1}{8}\cos(\tfrac{2}{3}\pi)--\tfrac{1}{8}\cos(0)=-\tfrac{1}{8}\cdot-\tfrac{1}{2}+\tfrac{1}{8}\cdot 1=\tfrac{1}{16}+\tfrac{1}{8}=\tfrac{3}{16}.$$



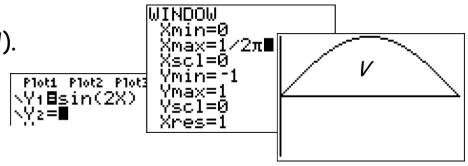
49b $\cos(2A)=1-2\sin^2(A) \Rightarrow \cos(2A)-1=-2\sin^2(A) \Rightarrow \frac{1}{4}\cos(2A)-\frac{1}{4}=-\frac{1}{2}\sin^2(A).$

$$\int_{\frac{1}{3}\pi}^{\pi}\left(2-\tfrac{1}{2}\sin^2(x)\right)dx=\int_{\frac{1}{3}\pi}^{\pi}\left(\tfrac{1}{4}\cos(\boxed{2x})+1\tfrac{3}{4}\right)dx=\left[\tfrac{1}{8}\sin(2x)+1\tfrac{3}{4}x\right]_{\frac{1}{3}\pi}^{\pi}=\tfrac{1}{8}\sin(2\pi)+1\tfrac{3}{4}\pi-\left(\tfrac{1}{8}\sin(\tfrac{2}{3}\pi)+1\tfrac{3}{4}\cdot\tfrac{1}{3}\pi\right)$$



$$=\tfrac{1}{8}\cdot 0+1\tfrac{3}{4}\pi-\left(\tfrac{1}{8}\cdot\tfrac{1}{2}\sqrt{3}+\tfrac{7}{12}\pi\right)=1\tfrac{3}{4}\pi-\tfrac{1}{16}\sqrt{3}-\tfrac{7}{12}\pi=\tfrac{7}{6}\pi-\tfrac{1}{16}\sqrt{3}.$$

50 $\cos(2A)=1-2\sin^2(A) \Rightarrow 2\sin^2(A)=1-\cos(2A) \Rightarrow \sin^2(A)=\frac{1}{2}-\frac{1}{2}\cos(2A).$



$$I(L)=\int_0^{\frac{1}{2}\pi}\pi\cdot(f(x))^2\,dx=\int_0^{\frac{1}{2}\pi}\pi\cdot\sin^2(2x)\,dx=\int_0^{\frac{1}{2}\pi}\pi\cdot\left(\tfrac{1}{2}-\tfrac{1}{2}\cos(\boxed{4x})\right)dx$$

$$=\left[\pi\cdot\left(\tfrac{1}{2}x-\tfrac{1}{8}\sin(4x)\right)\right]_0^{\frac{1}{2}\pi}=\pi\cdot\left(\tfrac{1}{4}\pi-\tfrac{1}{8}\sin(2\pi)\right)-\pi\cdot\left(0-\tfrac{1}{8}\sin(0)\right)=\pi\cdot\left(\tfrac{1}{4}\pi-0\right)-\pi\cdot(0-0)=\tfrac{1}{4}\pi^2.$$

51a $f(x)=2\left(\boxed{\sin(x)}\right)^2+\sin(x)-1 \Rightarrow f'(x)=4\sin(x)\cdot\cos(x)+\cos(x).$

$f'(x)=0 \Rightarrow 4\sin(x)\cdot\cos(x)+\cos(x)=0 \Rightarrow \cos(x)\cdot(4\sin(x)+1)=0$

$\cos(x)=0 \vee 4\sin(x)=-1 \Rightarrow x=\frac{1}{2}\pi+k\cdot\pi \vee \sin(x)=-\frac{1}{4} \Rightarrow x=\frac{1}{2}\pi \vee x=1\frac{1}{2}\pi \vee \sin(x)=-\frac{1}{4}.$

$x=\frac{1}{2}\pi \Rightarrow f(\frac{1}{2}\pi)=2\sin^2(\frac{1}{2}\pi)+\sin(\frac{1}{2}\pi)-1=2\cdot 1^2+1-1=2+0=2.$

$x=1\frac{1}{2}\pi \Rightarrow f(1\frac{1}{2}\pi)=2\sin^2(1\frac{1}{2}\pi)+\sin(1\frac{1}{2}\pi)-1=2\cdot(-1)^2+-1-1=2-2=0.$

$\sin(x)=-\frac{1}{4} \Rightarrow f(x)=2\cdot(-\frac{1}{4})^2+-\frac{1}{4}-1=2\cdot\frac{1}{16}-\frac{1}{4}-1=\frac{1}{8}-\frac{1}{4}-1=\frac{1}{8}-\frac{2}{8}-1=-1\frac{1}{8}.$

Randextreem: $f(0)=f(2\pi)=2\cdot 0^2+0-1=-1.$ Dus $B_f=[-1\frac{1}{8},\,2].$

51b $f(x)=0 \Rightarrow 2\sin^2(x)+\sin(x)-1=0$ (stel $\sin(x)=t$)

$2t^2+t-1=0 \Rightarrow D=b^2-4\cdot a\cdot c=1^2-4\cdot 2\cdot -1=1+8=9 \Rightarrow \sqrt{D}=3.$

$t=\sin(x)=\frac{-1\pm 3}{2\cdot 2} \Rightarrow t=\sin(x)=-1 \vee t=\sin(x)=\frac{1}{2}$ (met x op $[0,2\pi]$) $\Rightarrow x=1\frac{1}{2}\pi$ (zoeken we niet) $\vee\ x=\frac{1}{6}\pi \vee x=\frac{5}{6}\pi.$

$\cos(2A)=1-2\sin^2(A) \Rightarrow 2\sin^2(A)-1=-\cos(2A).$

$$\int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi}\left(2\sin^2(x)+\sin(x)-1\right)dx=\int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi}\left(-\cos(\boxed{2x})+\sin(x)\right)dx=\left[-\tfrac{1}{2}\sin(2x)-\cos(x)\right]_{\frac{1}{6}\pi}^{\frac{5}{6}\pi}$$

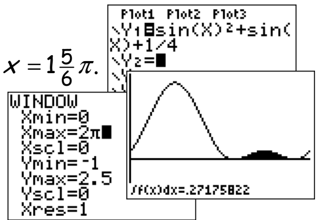
$$=-\tfrac{1}{2}\sin(\tfrac{5}{3}\pi)-\cos(\tfrac{5}{6}\pi)-\left(-\tfrac{1}{2}\sin(\tfrac{1}{3}\pi)-\cos(\tfrac{1}{6}\pi)\right)=-\tfrac{1}{2}\cdot-\tfrac{1}{2}\sqrt{3}--\tfrac{1}{2}\sqrt{3}-\left(-\tfrac{1}{2}\cdot\tfrac{1}{2}\sqrt{3}-\tfrac{1}{2}\sqrt{3}\right)$$

$$=\tfrac{1}{4}\sqrt{3}+\tfrac{1}{2}\sqrt{3}+\tfrac{1}{4}\sqrt{3}+\tfrac{1}{2}\sqrt{3}=1\tfrac{1}{2}\sqrt{3}.$$

52 $f(x)=0 \Rightarrow \sin^2(x)+\sin(x)+\frac{1}{4}=0 \Rightarrow \left(\sin(x)+\frac{1}{2}\right)^2=0 \Rightarrow \sin(x)=-\frac{1}{2} \Rightarrow x=1\frac{1}{6}\pi \vee x=1\frac{5}{6}\pi.$

$f(0)=0^2+0+\frac{1}{4}=\frac{1}{4}>0 \Rightarrow$ het ingesloten gebied loopt van $x=1\frac{1}{6}\pi$ tot $x=1\frac{5}{6}\pi.$

$\cos(2A)=1-2\sin^2(A) \Rightarrow 2\sin^2(A)=1-\cos(2A) \Rightarrow \sin^2(A)=\frac{1}{2}-\frac{1}{2}\cos(2A).$

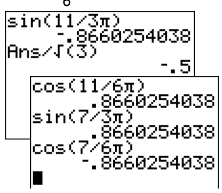


$$\int_{1\frac{1}{6}\pi}^{1\frac{5}{6}\pi}\left(\sin^2(x)+\sin(x)+\tfrac{1}{4}\right)dx=\int_{1\frac{1}{6}\pi}^{1\frac{5}{6}\pi}\left(\tfrac{1}{2}-\tfrac{1}{2}\cos(\boxed{2x})+\sin(x)+\tfrac{1}{4}\right)dx=\left[\tfrac{3}{4}x-\tfrac{1}{4}\sin(2x)-\cos(x)\right]_{1\frac{1}{6}\pi}^{1\frac{5}{6}\pi}$$

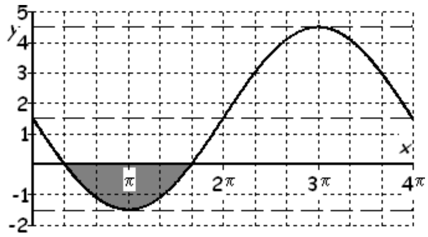
$$=\tfrac{3}{4}\cdot 1\tfrac{5}{6}\pi-\tfrac{1}{4}\sin(3\tfrac{2}{3}\pi)-\cos(1\tfrac{5}{6}\pi)-\left(\tfrac{3}{4}\cdot 1\tfrac{1}{6}\pi-\tfrac{1}{4}\sin(2\tfrac{1}{3}\pi)-\cos(1\tfrac{1}{6}\pi)\right)$$

$$=\tfrac{33}{24}\pi-\tfrac{1}{4}\cdot-\tfrac{1}{2}\sqrt{3}-\tfrac{1}{2}\sqrt{3}-\left(\tfrac{21}{24}\pi-\tfrac{1}{4}\cdot\tfrac{1}{2}\sqrt{3}--\tfrac{1}{2}\sqrt{3}\right)$$

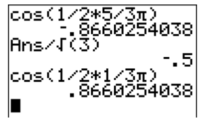
$$=\tfrac{33}{24}\pi+\tfrac{1}{8}\sqrt{3}-\tfrac{1}{2}\sqrt{3}-\tfrac{21}{24}\pi+\tfrac{1}{8}\sqrt{3}-\tfrac{1}{2}\sqrt{3}=\tfrac{12}{24}\pi+\tfrac{2}{8}\sqrt{3}-\sqrt{3}=\tfrac{1}{2}\pi-\tfrac{3}{4}\sqrt{3}.$$



53a $f(x)=1\frac{1}{2}-3\sin(\frac{1}{2}x)$ heeft
evenwichtsstand $1\frac{1}{2}$; amplitude 3; periode $\frac{2\pi}{\frac{1}{2}}=4\pi$ en beginpunt $(2\pi, 1\frac{1}{2})$.
Zie de grafiek van $f(x)=1\frac{1}{2}-3\sin(\frac{1}{2}x)$ hiernaast.

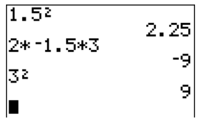


53b $f(x)=0 \Rightarrow 1\frac{1}{2}-3\sin(\frac{1}{2}x)=0 \Rightarrow -3\sin(\frac{1}{2}x)=-1\frac{1}{2} \Rightarrow \sin(\frac{1}{2}x)=\frac{1}{2} \Rightarrow$
$\frac{1}{2}x=\frac{1}{6}\pi+k\cdot 2\pi \vee \frac{1}{2}x=\frac{5}{6}\pi+k\cdot 2\pi \Rightarrow x=\frac{1}{3}\pi+k\cdot 4\pi \vee x=\frac{5}{3}\pi+k\cdot 4\pi.$
$x[0, 4\pi] \Rightarrow x=\frac{1}{3}\pi \vee x=\frac{5}{3}\pi.$ (het gevraagde gebied ligt ONDER de x-as)



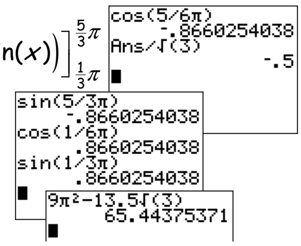
$$O(V)=\int_{\frac{1}{3}\pi}^{\frac{5}{3}\pi} -f(x)\,dx=\int_{\frac{1}{3}\pi}^{\frac{5}{3}\pi}\left(-1\frac{1}{2}+3\sin(\boxed{\tfrac{1}{2}x})\right)dx=\left[-1\frac{1}{2}x+\frac{3}{\frac{1}{2}}\cdot -\cos(\tfrac{1}{2}x)\right]_{\frac{1}{3}\pi}^{\frac{5}{3}\pi}=\left[-1\frac{1}{2}x-6\cos(\tfrac{1}{2}x)\right]_{\frac{1}{3}\pi}^{\frac{5}{3}\pi}$$
$$=-1\frac{1}{2}\cdot\frac{5}{3}\pi-6\cos(\tfrac{1}{2}\cdot\tfrac{5}{3}\pi)-\left(-1\frac{1}{2}\cdot\frac{1}{3}\pi-6\cos(\tfrac{1}{2}\cdot\tfrac{1}{3}\pi)\right)=-\frac{5}{2}\pi-6\cos(\tfrac{5}{6}\pi)-\left(-\frac{1}{2}\pi-6\cos(\tfrac{1}{6}\pi)\right)$$
$$=-2\frac{1}{2}\pi-6\cdot-\frac{1}{2}\sqrt{3}-\left(-\frac{1}{2}\pi-6\cdot\frac{1}{2}\sqrt{3}\right)=-2\frac{1}{2}\pi+3\sqrt{3}+\frac{1}{2}\pi+3\sqrt{3}=6\sqrt{3}-2\pi.$$

53c $$I(L)=\int_{\frac{1}{3}\pi}^{\frac{5}{3}\pi}\pi\cdot(f(x))^2\,dx=\int_{\frac{1}{3}\pi}^{\frac{5}{3}\pi}\pi\cdot\left(1\frac{1}{2}-3\sin(\tfrac{1}{2}x)\right)^2dx=\int_{\frac{1}{3}\pi}^{\frac{5}{3}\pi}\pi\cdot\left(2\frac{1}{4}-9\sin(\tfrac{1}{2}x)+9\sin^2(\tfrac{1}{2}x)\right)dx$$



Nu is: $\cos(2A)=1-2\sin^2(A) \Rightarrow 2\sin^2(A)=1-\cos(2A) \Rightarrow \sin^2(A)=\frac{1}{2}-\frac{1}{2}\cos(2A).$

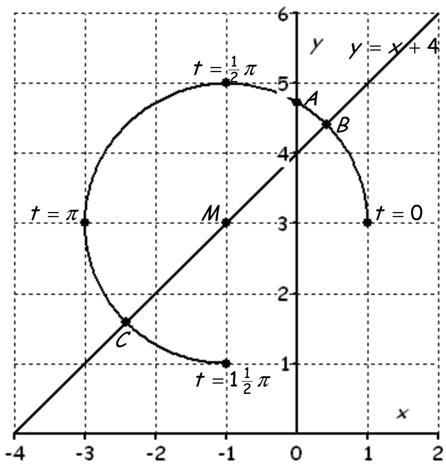
$$=\int_{\frac{1}{3}\pi}^{\frac{5}{3}\pi}\pi\cdot\left(2\frac{1}{4}-9\sin(\boxed{\tfrac{1}{2}x})+9\cdot\left(\frac{1}{2}-\frac{1}{2}\cos(x)\right)\right)dx$$
$$=\left[\pi\cdot\left(2\frac{1}{4}x-\frac{9}{\frac{1}{2}}\cdot-\cos(\tfrac{1}{2}x)+\frac{9}{2}x-\frac{9}{2}\sin(x)\right)\right]_{\frac{1}{3}\pi}^{\frac{5}{3}\pi}=\left[\pi\cdot\left(\frac{27}{4}x+18\cos(\tfrac{1}{2}x)-\frac{9}{2}\sin(x)\right)\right]_{\frac{1}{3}\pi}^{\frac{5}{3}\pi}$$
$$=\pi\cdot\left(\frac{27}{4}\cdot\frac{5}{3}\pi+18\cos(\tfrac{5}{6}\pi)-\frac{9}{2}\sin(\tfrac{5}{3}\pi)\right)-\pi\cdot\left(\frac{27}{4}\cdot\frac{1}{3}\pi+18\cos(\tfrac{1}{6}\pi)-\frac{9}{2}\sin(\tfrac{1}{3}\pi)\right)$$
$$=\pi\cdot\left(\frac{45}{4}\pi+18\cdot-\frac{1}{2}\sqrt{3}-\frac{9}{2}\cdot-\frac{1}{2}\sqrt{3}\right)-\pi\cdot\left(\frac{9}{4}\pi+18\cdot\frac{1}{2}\sqrt{3}-\frac{9}{2}\cdot\frac{1}{2}\sqrt{3}\right)$$
$$=11\frac{1}{4}\pi^2-9\pi\sqrt{3}+2\frac{1}{4}\pi\sqrt{3}-2\frac{1}{4}\pi^2-9\pi\sqrt{3}+2\frac{1}{4}\pi\sqrt{3}=9\pi^2-13\frac{1}{2}\pi\sqrt{3}.$$



54a $y_P=\sin(ct)$ en de periode is $5=\frac{2\pi}{c} \Rightarrow c=\frac{2\pi}{5}$.
Of voor $t=5$ is $y_P=\sin(\frac{2\pi}{5}\cdot 5)=\sin(2\pi)$
(de sinus heeft dan precies één periode doorlopen)

54b Formule II: $x_P=\cos(\frac{2\pi}{5}t)$.

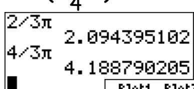
55 $\text{rc}_{y=-x+3}=-1 \Rightarrow \angle AMB=45^\circ \Rightarrow$ (ΔAMB is een 1-1-$\sqrt{2}$ driehoek) $AB=MB$.
$AM=4 \Rightarrow AB=MB=\frac{4}{\sqrt{2}}=\frac{4}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{4\sqrt{2}}{2}=2\sqrt{2}.$
$x_A=x_M-AB=2-2\sqrt{2}$ en $y_A=y_M+AB=1+2\sqrt{2}.$



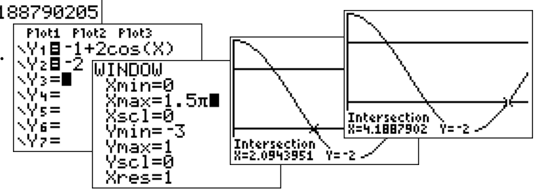
56a $t=0 \Rightarrow P(1, 3)$. P draait linksom. t op $[0, 1\frac{1}{2}\pi] \Rightarrow$ driekwartcirkel.
De baan van P is driekwartcirkel met middelpunt $M(-1, 3)$ en straal 2.

56b $x=0 \Rightarrow -1+2\cos(t)=0 \Rightarrow 2\cos(t)=1 \Rightarrow \cos(t)=\frac{1}{2}$ (t op $[0, 1\frac{1}{2}\pi]$) $\Rightarrow t=\frac{1}{3}\pi.$
$t=\frac{1}{3}\pi \Rightarrow y_A=3+2\sin(\frac{1}{3}\pi)=3+2\cdot\frac{1}{2}\sqrt{3}=3+\sqrt{3} \Rightarrow A(0, 3+\sqrt{3}).$

56c $\text{rc}_{y=x+4}=1 \Rightarrow$ bij B hoort $t=\frac{1}{4}\pi$ en bij C hoort $t=1\frac{1}{4}\pi$.
$t=\frac{1}{4}\pi \Rightarrow x_B=-1+2\cos(\frac{1}{4}\pi)=-1+\sqrt{2}$ en $y_B=3+2\sin(\frac{1}{4}\pi)=3+\sqrt{2}.$
$t=1\frac{1}{4}\pi \Rightarrow x_C=-1+2\cos(1\frac{1}{4}\pi)=-1-\sqrt{2}$ en $y_C=3+2\sin(1\frac{1}{4}\pi)=3-\sqrt{2}.$

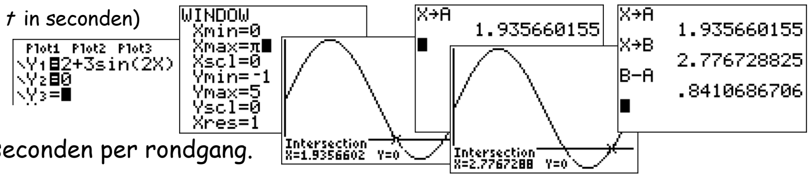


56d $x=-2 \Rightarrow -1+2\cos(t)=-2$ (intersect of) $\Rightarrow$
$2\cos(t)=-1 \Rightarrow \cos(t)=-\frac{1}{2} \Rightarrow t=\frac{2}{3}\pi+k\cdot 2\pi \vee t=-\frac{2}{3}\pi+k\cdot 2\pi.$
t op $[0, 1\frac{1}{2}\pi] \Rightarrow t=\frac{2}{3}\pi \approx 2{,}09 \vee t=\frac{4}{3}\pi \approx 4{,}19.$
$x<-2$ (zie driekwartcirkel of plot) $\Rightarrow 2{,}09<t<4{,}19.$

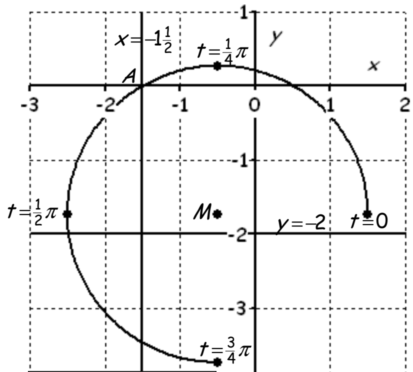


57a $x_P = 5 + 3\cos(2t)$ en $y_P = 2 + 3\sin(2t)$. (met t in seconden)

57b De eerste rondgang van $t = 0$ tot $t = \pi$.
$y = 0$ (x-as) $\Rightarrow 2 + 3\sin(2t) = 0$ (intersect) $\Rightarrow$
$t \approx 1{,}94 \vee t \approx 2{,}78$.
$y < 0$ (zie plot) $\Rightarrow 1{,}94 < t < 2{,}78$. Dus 0,84 seconden per rondgang.



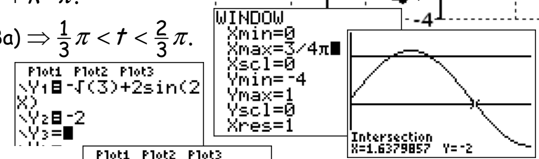
58a $t = 0 \Rightarrow P(1\frac{1}{2}, -\sqrt{3})$. P draait linksom. t op $[0, \frac{3}{4}\pi] \Rightarrow 2t$ op $[0, 1\frac{1}{2}\pi]$.
De baan van P is driekwartcirkel met middelpunt $M(-\frac{1}{2}, -\sqrt{3})$ en straal 2.



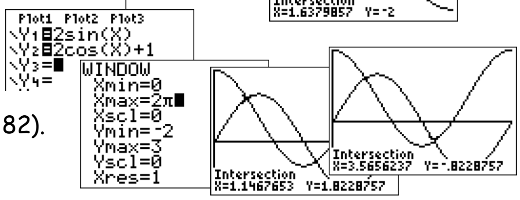
58b $y = 0$ (x-as) $\Rightarrow -\sqrt{3} + 2\sin(2t) = 0 \Rightarrow 2\sin(2t) = \sqrt{3} \Rightarrow \sin(2t) = \frac{1}{2}\sqrt{3} \Rightarrow$
$2t = \frac{1}{3}\pi + k \cdot 2\pi \vee 2t = \pi - \frac{1}{3}\pi + k \cdot 2\pi$
$t = \frac{1}{6}\pi + k \cdot \pi \vee t = \frac{1}{3}\pi$ (deze zoeken we) $+ k \cdot \pi$.
$x_A = -\frac{1}{2} + 2\cos(\frac{2}{3}\pi) = -\frac{1}{2} + 2 \cdot -\frac{1}{2} = -1\frac{1}{2} \Rightarrow A(-1\frac{1}{2}, 0)$.

58c $x = -1\frac{1}{2} \Rightarrow -\frac{1}{2} + 2\cos(2t) = -1\frac{1}{2} \Rightarrow 2\cos(2t) = -1 \Rightarrow \cos(2t) = -\frac{1}{2} \Rightarrow$
$2t = \frac{2}{3}\pi + k \cdot 2\pi \vee 2t = -\frac{2}{3}\pi + k \cdot 2\pi \Rightarrow t = \frac{1}{3}\pi + k \cdot \pi \vee t = -\frac{1}{3}\pi + k \cdot \pi$.
t op $[0, \frac{3}{4}\pi] \Rightarrow t = \frac{1}{3}\pi \vee t = \frac{2}{3}\pi$. Dus $x < -1\frac{1}{2}$ (zie de baan bij 58a) $\Rightarrow \frac{1}{3}\pi < t < \frac{2}{3}\pi$.

58d $y = -2 \Rightarrow -\sqrt{3} + 2\sin(2t) = -2$ (intersect) $\Rightarrow t \approx 1{,}64$.
Dus $y < -2$ (zie 58a of de plot) $\Rightarrow 1{,}64 < t \le \frac{3}{4}\pi$.



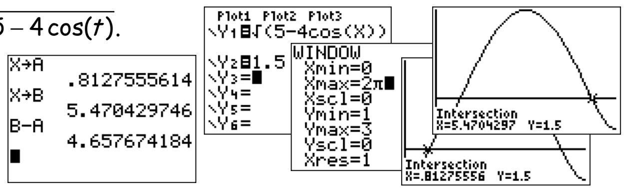
59a $y = x + 1 \Rightarrow 2\sin(t) = 2\cos(t) + 1$ (met $0 \le t < 2\pi$) intersect geeft dan
$t \approx 1{,}15$ en $y \approx 1{,}82 \Rightarrow x = y - 1 = 0{,}82 \Rightarrow$ snijpunt $(0{,}82; 1{,}82)$
of $t \approx 3{,}57$ en $y \approx -0{,}82 \Rightarrow x = y - 1 = -1{,}82 \Rightarrow$ snijpunt $(-1{,}82; -0{,}82)$.



59b $x = 1 \Rightarrow 2\cos(t) = 1 \Rightarrow \cos(t) = \frac{1}{2} \Rightarrow t = \frac{1}{3}\pi + k \cdot 2\pi \vee t = -\frac{1}{3}\pi + k \cdot 2\pi$.
Er geldt nu: $x > 1$ voor $-\frac{1}{3}\pi < t < \frac{1}{3}\pi$. De baan van P is een cirkel met middelpunt $(0, 0)$ en straal 2.
Dus ligt $\frac{\frac{2}{3}\pi}{2\pi} = \frac{1}{3}$ deel van de cirkel rechts van de lijn $x = 1$. (omtrek van een cirkel $= 2\pi r$)
De lengte van het deel rechts van de lijn $x = 1$ is $\frac{1}{3} \cdot 2\pi \cdot 2 = \frac{4}{3}\pi$.

59c $P(2\cos(t), 2\sin(t))$ en $Q(\cos(2t), \sin(2t))$. Nu de stelling van Pythagoras:
$PQ^2 = (\cos(2t) - 2\cos(t))^2 + (\sin(2t) - 2\sin(t))^2$
$= \cos^2(2t) - 4\cos(2t)\cos(t) + 4\cos(t)^2 + \sin^2(2t) - 4\sin(2t)\sin(t) + 4\sin^2(t)$
$= \cos^2(2t) + \sin^2(2t) + 4\cos(t)^2 + 4\sin^2(t) - 4\cos(2t)\cos(t) - 4\sin(2t)\sin(t)$
$= \cos^2(2t) + \sin^2(2t) + 4 \cdot \left(\cos(t)^2 + \sin^2(t)\right) - 4 \cdot (\cos(2t)\cos(t) + \sin(2t)\sin(t))$
$= 1 + 4 \cdot 1 - 4 \cdot \cos(2t - t) = 5 - 4\cos(t)$. Dus $PQ = \sqrt{5 - 4\cos(t)}$.

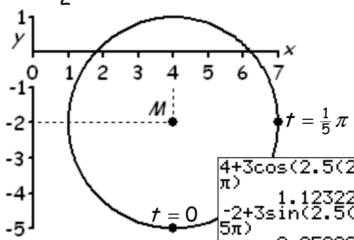
59d $PQ = \sqrt{5 - 4\cos(t)} = 1\frac{1}{2}$ (intersect) $\Rightarrow t \approx 0{,}81 \vee t \approx 5{,}47$.
$PQ = \sqrt{5 - 4\cos(t)} > 1\frac{1}{2}$ (zie plot) $\Rightarrow 0{,}81 < t < 5{,}47$.
Dus gedurende 4,66 seconde per rondgang.



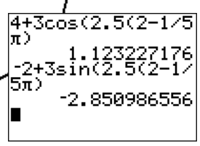
60a De translatie $(2, 0)$.

60b $x_Q = 3\cos(\frac{1}{2}(t - 2))$ en $y_Q = 3\sin(\frac{1}{2}(t - 2))$. (met t in seconden)

61a De omlooptijd $T = \frac{2\pi}{2\frac{1}{2}} = \frac{4\pi}{5} = \frac{4}{5}\pi$ seconden.
Na $\frac{1}{4} \cdot \frac{4}{5}\pi = \frac{1}{5}\pi$ seconde (voor $t = \frac{1}{5}\pi$) bevindt P zich in $(7, -2)$.
Dus $x_P = 4 + 3\cos(2\frac{1}{2}(t - \frac{1}{5}\pi))$ en $y_P = -2 + 3\sin(2\frac{1}{2}(t - \frac{1}{5}\pi))$. (met t in seconden)

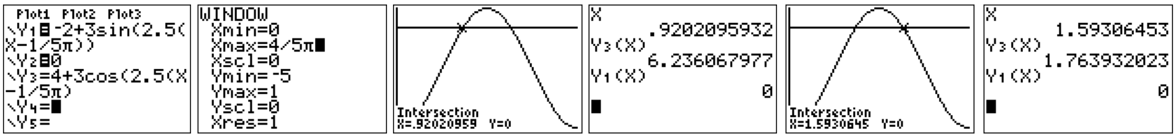


61b $t = 2 \Rightarrow x_P = 4 + 3\cos(2\frac{1}{2}(2 - \frac{1}{5}\pi)) \approx 1{,}12$ en $y_P = -2 + 3\sin(2\frac{1}{2}(2 - \frac{1}{5}\pi)) \approx -2{,}85$.

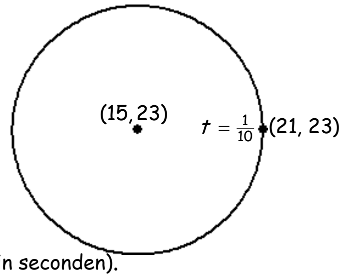


61c Na $\frac{3}{4} \cdot \frac{4}{5}\pi = \frac{3}{5}\pi$ seconde (voor $t = \frac{3}{5}\pi$) bevindt P zich voor het eerst in $(1, -2)$.

61d $y = 0$ (x-as) $\Rightarrow -2 + 3\sin\left(2\frac{1}{2}(t - \frac{1}{5}\pi)\right) = 0$ (intersect)
$t \approx 0{,}92 \Rightarrow x_P = 4 + 3\cos\left(2\frac{1}{2}(t - \frac{1}{5}\pi)\right) \approx 6{,}24 \Rightarrow$ snijpunt met x-as (6,24; 0).
$t \approx 1{,}59 \Rightarrow x_P = 4 + 3\cos\left(2\frac{1}{2}(t - \frac{1}{5}\pi)\right) \approx 1{,}76 \Rightarrow$ snijpunt met x-as (1,76; 0).



62a $\begin{cases} x_P = 15 + 6\cos\left(4\pi(t - \frac{1}{10})\right) \\ y_P = 23 + 6\sin\left(4\pi(t - \frac{1}{10})\right) \end{cases}$ (t in seconden). $\omega = \frac{2\pi}{\frac{1}{2}} = \frac{4\pi}{1} = 4\pi$

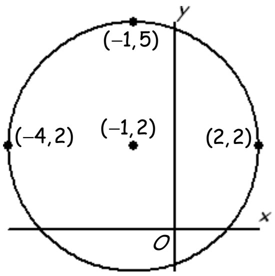


62b $\begin{cases} x_Q = 15 + 6\cos\left(4\pi(t + \frac{1}{5} - \frac{1}{10})\right) = 15 + 6\cos\left(4\pi(t + \frac{1}{10})\right) \\ y_Q = 23 + 6\sin\left(4\pi(t + \frac{1}{5} - \frac{1}{10})\right) = 23 + 6\sin\left(4\pi(t + \frac{1}{10})\right) \end{cases}$ (t in seconden).

62c $\begin{cases} x_R = 15 + 6\cos(4\pi t - \pi) = 15 + 6\cos\left(4\pi(t - \frac{1}{4})\right) = 15 + 6\cos\left(4\pi(t - \frac{1}{10} - \frac{3}{20})\right) \\ y_R = 23 + 6\sin(4\pi t - \pi) = 23 + 6\sin\left(4\pi(t - \frac{1}{4})\right) = 23 + 6\sin\left(4\pi(t - \frac{1}{10} - \frac{3}{20})\right) \end{cases}$ (t in seconden).
Dus R loopt $\frac{3}{20}$ seconde achter op P of (omdat de omlooptijd $\frac{1}{2}$ seconde is) $\frac{7}{20}$ seconde voor op P.

63a Omlooptijd is $\frac{2\pi}{4} = \frac{1}{2}\pi$ seconde.
$\begin{cases} x_P = -1 + 3\cos(4t) \\ y_P = 2 + 3\sin(4t) \end{cases}$ en $\begin{cases} x_Q = -1 + 3\cos\left(4 \cdot (t - \frac{1}{3} \cdot \frac{1}{2}\pi)\right) = -1 + 3\cos\left(4 \cdot (t - \frac{1}{6}\pi)\right) \\ y_Q = 2 + 3\sin\left(4 \cdot (t - \frac{1}{3} \cdot \frac{1}{2}\pi)\right) = 2 + 3\sin\left(4 \cdot (t - \frac{1}{6}\pi)\right) \end{cases}$ (t in seconden).

63b Op $t = 0$ heeft P een fasevoorsprong van $\frac{1}{4}$ op (2, 2).
Q heeft een fasevoorsprong van $\frac{1}{6}$ op $P \Rightarrow$ fasevoorsprong van Q op (2, 2) is $\frac{1}{4} + \frac{1}{6} = \frac{5}{12}$.
$\begin{cases} x_Q = -1 + 3\cos\left(4 \cdot (t + \frac{5}{12} \cdot \frac{1}{2}\pi)\right) = -1 + 3\cos\left(4 \cdot (t + \frac{5}{24}\pi)\right) \\ y_Q = 2 + 3\sin\left(4 \cdot (t + \frac{5}{12} \cdot \frac{1}{2}\pi)\right) = 2 + 3\sin\left(4 \cdot (t + \frac{5}{24}\pi)\right) \end{cases}$ (t in seconden).



63c Op $t = 0$ heeft P een fasevoorsprong van $\frac{1}{2}$ op (2, 2).
Q heeft een faseachterstand van $\frac{1}{4}$ op $P \Rightarrow$ fasevoorsprong van Q op (2, 2) is $\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$.
$\begin{cases} x_Q = -1 + 3\cos\left(4 \cdot (t + \frac{1}{4} \cdot \frac{1}{2}\pi)\right) = -1 + 3\cos\left(4 \cdot (t + \frac{1}{8}\pi)\right) \\ y_Q = 2 + 3\sin\left(4 \cdot (t + \frac{1}{4} \cdot \frac{1}{2}\pi)\right) = 2 + 3\sin\left(4 \cdot (t + \frac{1}{8}\pi)\right) \end{cases}$ (t in seconden).

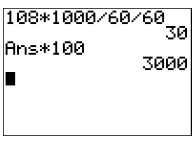
64a De fasevoorsprong van Q op P is $\frac{\frac{2}{3}\pi}{2\pi} = \frac{1}{3}$.

64b De faseachterstand van R op P is $\frac{\frac{1}{2}\pi}{2\pi} = \frac{1}{4}$.

64c De fasevoorsprong van Q op R is $\frac{1}{3} + \frac{1}{4} = \frac{7}{12} (> \frac{1}{2}) \Rightarrow$ faseverschil tussen Q en R is $1 - \frac{7}{12} = \frac{5}{12}$.

65a Omlooptijd in stand I is $\frac{1}{15}$ seconde $\Rightarrow \omega = \frac{2\pi}{\frac{1}{15}} = 2\pi \cdot 15 = 30\pi$ rad/sec.
Q heeft een faseachterstand van $\frac{1}{3}$ op P
$\begin{cases} x_P = 20\cos(30\pi t) \\ y_P = 20\sin(30\pi t) \end{cases}$ en $\begin{cases} x_Q = 20\cos\left(30\pi \cdot (t - \frac{1}{3} \cdot \frac{1}{15})\right) = 20\cos\left(30\pi \cdot (t - \frac{1}{45})\right) \\ y_Q = 20\sin\left(30\pi \cdot (t - \frac{1}{3} \cdot \frac{1}{15})\right) = 20\sin\left(30\pi \cdot (t - \frac{1}{45})\right) \end{cases}$ (t in seconden).
R heeft een fasevoorsprong van $\frac{1}{3}$ op P, dus $\begin{cases} x_R = 20\cos\left(30\pi \cdot (t + \frac{1}{45})\right) \\ y_R = 20\sin\left(30\pi \cdot (t + \frac{1}{45})\right) \end{cases}$ (t in seconden).

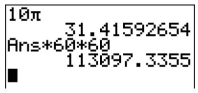
65b $v = 108$ km/uur $= 30$ m/s $= 3000$ cm/s.
Omlooptijd bij II is $\frac{2\pi \cdot 20}{3000} = \frac{4\pi}{300}$ sec.
Dus $\omega = 2\pi : \frac{4\pi}{300} = 2\pi \cdot \frac{300}{4\pi} = 150$ rad/sec.
$\begin{cases} x_P = 20\cos(150t) \\ y_P = 20\sin(150t) \end{cases}$ (t in seconden).



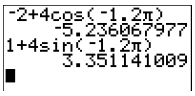
66a De diameter van rol II is de helft van rol I, dus de omlooptijd van rol II is de helft van de omlooptijd van rol I.

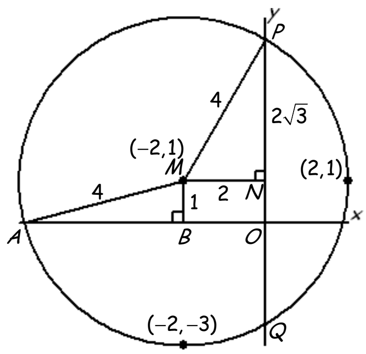
$\begin{cases} x_P = 10\cos(-\frac{2\pi}{2}t) = 10\cos(\pi t) \\ y_P = 10\sin(-\frac{2\pi}{2}t) = -10\sin(\pi t) \end{cases}$ en $\begin{cases} x_Q = 5\cos(2\pi t + \pi) \\ y_Q = 5\sin(2\pi t + \pi) \end{cases}$ (t in seconden).

66b Omlooptijd rol II is 1 sec. $\Rightarrow$ elke seconde loopt $2\pi \cdot 5 = 10\pi$ cm papier tussen de rollen door.
Dat is per uur $10\pi \cdot 60 \cdot 60 = 36\,000\pi$ cm $\approx 113\,097$ cm ≈ 1131 m.



67a $\begin{cases} x_P = -2 + 4\cos(-\pi t) \\ y_P = 1 + 4\sin(-\pi t) \end{cases}$ (t in seconden).



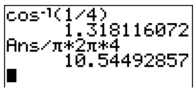


67b $t = 1{,}2 \Rightarrow x_P = -2 + 4\cos(-1{,}2\pi) \approx -5{,}24$ en $y_P = 1 + 4\sin(-1{,}2\pi) \approx 3{,}35$.

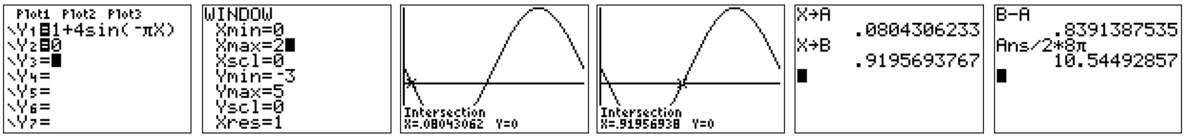
67c De baan wordt in negatieve richting (met de wijzers van de klok) doorlopen.
$\frac{1}{4}$ periode is $\frac{1}{4} \cdot 2 = \frac{1}{2}$ seconde $\Rightarrow t = \frac{1}{2}$, $t = 2\frac{1}{2}$ en $t = 4\frac{1}{2}$.

67d $x = 0$ (y-as) $\Rightarrow -2 + 4\cos(-\pi t) = 0 \Rightarrow 4\cos(-\pi t) = 2 \Rightarrow \cos(-\pi t) = \frac{1}{2} \Rightarrow$
$-\pi t = \frac{1}{3}\pi + k \cdot 2\pi \vee -\pi t = -\frac{1}{3}\pi + k \cdot 2\pi \Rightarrow t = -\frac{1}{3} + k \cdot 2 \vee t = \frac{1}{3} + k \cdot 2$.
$t = -\frac{1}{3} \Rightarrow y = 1 + 4\sin(\frac{1}{3}\pi) = 1 + 4 \cdot \frac{1}{2}\sqrt{3} = 1 + 2\sqrt{3} \Rightarrow P(0, 1 + 2\sqrt{3})$.
$t = \frac{1}{3} \Rightarrow y = 1 + 4\sin(-\frac{1}{3}\pi) = 1 - 4 \cdot \frac{1}{2}\sqrt{3} = 1 - 2\sqrt{3} \Rightarrow Q(0, 1 - 2\sqrt{3})$.
Alternatieve uitwerking: $NP^2 = 4^2 - 2^2 = 12 \Rightarrow NP = \sqrt{12} = 2\sqrt{3} \Rightarrow P(0, 1 + 2\sqrt{3})$ en $Q(0, 1 - 2\sqrt{3})$.

67e $\cos\angle AMB = \frac{1}{4} \Rightarrow \angle AMB \approx 1{,}318$ (rad)
De lengte van de boog onder de x-as is $\frac{2 \cdot Ans}{2\pi} \cdot 2\pi \cdot 4 \approx 10{,}54$.



Alternatieve uitwerking:
$y = 0$ (x-as) $\Rightarrow 1 + 4\sin(-\pi t) = 0$ (intersect) $\Rightarrow t \approx 0{,}08 \vee t \approx 0{,}92$.
$y < 0$ (zie plot) $\Rightarrow 0{,}08 < t < 0{,}92$.
Dus ongeveer 0,84 seconden van de 2 seconden per omwenteling onder de x-as.
De lengte van de boog onder de x-as is $\frac{Ans}{2} \cdot 2\pi \cdot 4 \approx 10{,}54$.



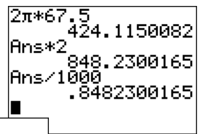
68a $T = 30 \Rightarrow \omega = \frac{2\pi}{30} = \frac{1}{15}\pi$ rad/min. Op $t = 0$ zit Frits in het hoogste punt.

Voor Frits geldt: $\begin{cases} x_{\text{Frits}} = 67\frac{1}{2}\cos\left(\frac{1}{15}\pi \cdot (t + \frac{1}{4} \cdot 30)\right) = 67\frac{1}{2}\cos(\frac{1}{15}\pi t + \frac{1}{2}\pi) \\ y_{\text{Frits}} = 67\frac{1}{2} + 67\frac{1}{2}\sin\left(\frac{1}{15}\pi \cdot (t + \frac{1}{4} \cdot 30)\right) = 67\frac{1}{2} + 67\frac{1}{2}\sin(\frac{1}{15}\pi t + \frac{1}{2}\pi) \end{cases}$ (t in minuten).

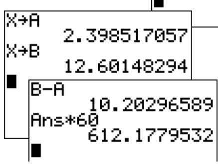
68b Saskia heeft $\frac{4}{32} = \frac{1}{8}$ faseachterstand op Frits.

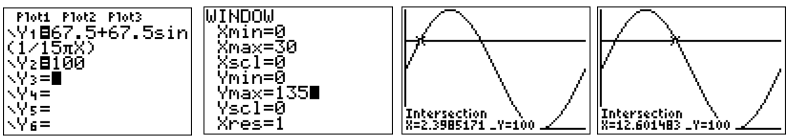
$\begin{cases} x_{\text{Saskia}} = 67\frac{1}{2}\cos\left(\frac{1}{15}\pi \cdot (t + \frac{1}{4} \cdot 30 - \frac{1}{8} \cdot 30)\right) = 67\frac{1}{2}\cos\left(\frac{1}{15}\pi \cdot (t + \frac{1}{8} \cdot 30)\right) \\ y_{\text{Saskia}} = 67\frac{1}{2} + 67\frac{1}{2}\sin\left(\frac{1}{15}\pi \cdot (t + \frac{1}{4} \cdot 30 - \frac{1}{8} \cdot 30)\right) = 67\frac{1}{2} + 67\frac{1}{2}\sin\left(\frac{1}{15}\pi \cdot (t + \frac{1}{8} \cdot 30)\right) \end{cases}$ (t in minuten).

68c De omtrek van het reuzenrad wordt afgelegd in 30 minuten
Dus $2\pi \cdot 67\frac{1}{2}$ meter in 30 minuten $\Rightarrow$ 848 m/uur $\Rightarrow$ de snelheid is ongeveer 0,85 km/uur.

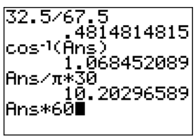


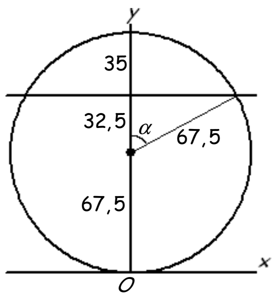
68d $y = 100 \Rightarrow 67\frac{1}{2} + 67\frac{1}{2}\sin(\frac{1}{15}\pi t) = 100$ (intersect) $\Rightarrow t \approx 2{,}40 \vee t \approx 12{,}60$
$y > 100$ (zie plot) $\Rightarrow 2{,}40 < t < 12{,}60$.
Dus gedurende ongeveer 10,2 minuten $\approx$ 612 seconden boven 100 meter.





Alternatieve uitwerking:
$\cos(\alpha) = \frac{32{,}5}{67{,}5} \Rightarrow \alpha \approx 1{,}068$ (rad).
Dus gedurende $\frac{2\alpha}{2\pi} \cdot 30 \approx 10{,}2$ min. ≈ 612 sec. boven 100 meter.





Diagnostische toets

D1a $-\cos(3x-\frac{1}{4}\pi)=\cos(3x-\frac{1}{4}\pi+\pi)=\cos(3x+\frac{3}{4}\pi)=\sin(3x+\frac{3}{4}\pi+\frac{1}{2}\pi)=\sin(3x+1\frac{1}{4}\pi).$

D1b $(\sin(x)+\cos(x))^2=\sin^2(x)+2\sin(x)\cos(x)+\cos^2(x)=\sin^2(x)+\cos^2(x)+2\sin(x)\cos(x)=1+\sin(2x).$

D1c $2+\cos(x)-2\sin^2(x)=2+\cos(x)-2\left(1-\cos^2(x)\right)=2+\cos(x)-2+2\cos^2(x)=2\cos^2(x)+\cos(x).$

D2a
$\sin(3x-\frac{1}{4}\pi)=\cos(2x)$
$\cos(3x-\frac{1}{4}\pi-\frac{1}{2}\pi)=\cos(2x)$
$3x-\frac{3}{4}\pi=2x+k\cdot 2\pi \vee 3x-\frac{3}{4}\pi=-2x+k\cdot 2\pi$
$x=\frac{3}{4}\pi+k\cdot 2\pi \vee 5x=\frac{3}{4}\pi+k\cdot 2\pi$
$x=\frac{3}{4}\pi+k\cdot 2\pi \vee x=\frac{3}{20}\pi+k\cdot\frac{2}{5}\pi$
x op $[0,\pi]\Rightarrow x=\frac{3}{4}\pi \vee x=\frac{3}{20}\pi \vee x=\frac{11}{20}\pi \vee x=\frac{19}{20}\pi.$

D2c
$\cos(\frac{2}{5}\pi t)=-\sin(\frac{1}{6}\pi t)$
$\sin(\frac{2}{5}\pi t+\frac{1}{2}\pi)=\sin(\frac{1}{6}\pi t+\pi)$
$\frac{2}{5}\pi t+\frac{1}{2}\pi=\frac{1}{6}\pi t+\pi+k\cdot 2\pi \vee \frac{2}{5}\pi t+\frac{1}{2}\pi=\pi-\frac{1}{6}\pi t-\pi+k\cdot 2\pi$
$\frac{7}{30}\pi t=\frac{1}{2}\pi+k\cdot 2\pi \vee \frac{17}{30}\pi t=-\frac{1}{2}\pi+k\cdot 2\pi$
$t=\frac{15}{7}+k\cdot\frac{60}{7} \vee t=-\frac{15}{17}+k\cdot\frac{60}{7}$
t op $[0,10]\Rightarrow t=\frac{15}{7} \vee t=\frac{45}{17} \vee t=\frac{105}{17} \vee t=\frac{165}{17}.$

D2b
$2\sin^2(2x)=\sin(2x)+1$
$-\sin(2x)=1-2\sin^2(2x)$
$\sin(2x+\pi)=\cos(4x)$
$\cos(2x+\pi-\frac{1}{2}\pi)=\cos(4x)$
$2x+\frac{1}{2}\pi=4x+k\cdot 2\pi \vee 2x+\frac{1}{2}\pi=-4x+k\cdot 2\pi$
$-2x=-\frac{1}{2}\pi+k\cdot 2\pi \vee 6x=-\frac{1}{2}\pi+k\cdot 2\pi$
$x=\frac{1}{4}\pi+k\cdot\pi \vee x=-\frac{1}{12}\pi+k\cdot\frac{1}{3}\pi$
x op $[0,2\pi]\Rightarrow x=\frac{1}{4}\pi \vee x=1\frac{1}{4}\pi \vee x=\frac{7}{12}\pi \vee x=\frac{11}{12}\pi \vee x=\frac{19}{12}\pi \vee x=\frac{23}{12}\pi.$

D3a
$\sin(x+\frac{1}{3}\pi)=2\sin(2x)\cdot\cos(2x)$
$\sin(x+\frac{1}{3}\pi)=\sin(4x)$
$x+\frac{1}{3}\pi=4x+k\cdot 2\pi \vee x+\frac{1}{3}\pi=\pi-4x+k\cdot 2\pi$
$-3x=-\frac{1}{3}\pi+k\cdot 2\pi \vee 5x=\frac{2}{3}\pi+k\cdot 2\pi$
$x=\frac{1}{9}\pi+k\cdot\frac{2}{3}\pi \vee x=\frac{2}{15}\pi+k\cdot\frac{2}{5}\pi.$

D3b
$\sin^2(2x)+\frac{1}{4}=\cos(4x)$
$\sin^2(2x)+\frac{1}{4}=1-2\sin^2(2x)$
$3\sin^2(2x)=\frac{3}{4}$
$\sin^2(2x)=\frac{1}{4}$
$\sin 2x=\pm\frac{1}{2}$
$2x=\frac{1}{6}\pi+k\cdot\pi \vee 2x=-\frac{1}{6}\pi+k\cdot\pi$
$x=\frac{1}{12}\pi+k\cdot\frac{1}{2}\pi \vee x=-\frac{1}{12}\pi+k\cdot\frac{1}{2}\pi.$

D4 $\tan(2x)=\dfrac{\sin(2x)}{\cos(2x)}=\dfrac{2\sin(x)\cos(x)}{\cos^2(x)-\sin^2(x)}=\dfrac{2\sin(x)\cos(x)}{\cos^2(x)-\sin^2(x)}\cdot\dfrac{\frac{1}{\cos^2(x)}}{\frac{1}{\cos^2(x)}}=\dfrac{\frac{2\sin(x)\cos(x)}{\cos^2(x)}}{\frac{\cos^2(x)}{\cos^2(x)}-\frac{\sin^2(x)}{\cos^2(x)}}=\dfrac{\frac{2\sin(x)}{\cos(x)}}{1-\left(\frac{\sin(x)}{\cos(x)}\right)^2}=\dfrac{2\tan(x)}{1-\tan^2(x)}.$

D5 $f(1\frac{1}{2}\pi-p)+f(1\frac{1}{2}\pi+p)=\sin(3\pi-2p)+\cos(1\frac{1}{2}\pi-p)+\sin(3\pi+2p)+\cos(1\frac{1}{2}\pi+p)$
$=\sin(2p)+\sin(p)-\sin(2p)-\sin(p)=0\Rightarrow f$ is symmetrisch in het punt $(1\frac{1}{2}\pi,0).$

D6a $f(x)=\cos(\boxed{2x})+\sin(\boxed{2x})\Rightarrow f'(x)=-2\sin(2x)+2\cos(2x).$

D6b $f(x)=2\cos^3(x)=2\left(\boxed{\cos(x)}\right)^3\Rightarrow f'(x)=2\cdot 3\left(\cos(x)\right)^2\cdot -\sin(x)=-6\sin(x)\cos^2(x).$

D6c $f(x)=\dfrac{\cos(x)}{\sin(x)}\Rightarrow f'(x)=\dfrac{\sin(x)\cdot -\sin(x)-\cos(x)\cdot\cos(x)}{\sin^2(x)}=\dfrac{-\sin^2(x)-\cos^2(x)}{\sin^2(x)}=\dfrac{-1}{\sin^2(x)}.$

D6d $f(x)=x^2\cdot\sin(\boxed{2x-\frac{1}{2}\pi})\Rightarrow f'(x)=2x\cdot\sin(2x-\frac{1}{2}\pi)+x^2\cdot 2\cos(2x-\frac{1}{2}\pi)=2x\sin(2x-\frac{1}{2}\pi)+2x^2\cos(2x-\frac{1}{2}\pi).$

D6e $f(x)=\sin(x)\cdot\tan(\boxed{2x})\Rightarrow f'(x)=\cos(x)\cdot\tan(2x)+\sin(x)\cdot\dfrac{1}{\cos^2(2x)}\cdot 2=\cos(x)\cdot\tan(2x)+\dfrac{2\sin(x)}{\cos^2(2x)}.$

D6f $f(x)=\dfrac{\tan(\boxed{2x})}{\sin(x)}\Rightarrow f'(x)=\dfrac{\sin(x)\cdot\frac{1}{\cos^2(2x)}\cdot 2-\tan(2x)\cdot\cos(x)}{\sin^2(x)}=\dfrac{\frac{2\sin(x)}{\cos^2(2x)}-\frac{\sin(2x)}{\cos(2x)}\cdot\cos(x)}{\sin^2(x)}\cdot\dfrac{\cos^2(2x)}{\cos^2(2x)}$
$=\dfrac{2\sin(x)-\sin(2x)\cos(2x)\cos(x)}{\sin^2(x)\cos^2(2x)}.$

OF ..... $f(x)=\dfrac{\tan(\boxed{2x})}{\sin(x)}\Rightarrow f'(x)=\dfrac{\sin(x)\cdot\left(1+\tan^2(2x)\right)\cdot 2-\tan(2x)\cdot\cos(x)}{\sin^2(x)}=\dfrac{2\sin(x)+2\sin(x)\tan^2(2x)-\tan(2x)\cos(x)}{\sin^2(x)}.$

D7a $f(x)=3-2\sin(x-\frac{1}{6}\pi)$ heeft evenwichtsstand 3; amplitude 2; periode $\frac{2\pi}{1}=2\pi$ en beginpunt $(\frac{1}{6}\pi+\pi,3)=(1\frac{1}{6}\pi,3)$.
Hoogste punten zijn $(\frac{7}{6}\pi+\frac{1}{4}\cdot 2\pi+k\cdot 2\pi,\ 3+2)=(\frac{5}{3}\pi+k\cdot 2\pi,\ 5)$.
Laagste punten zijn $(\frac{5}{3}\pi+\frac{1}{2}\cdot 2\pi+k\cdot 2\pi,\ 3-2)=(\frac{8}{3}\pi+k\cdot 2\pi,\ 1)=(\frac{2}{3}\pi+k\cdot 2\pi,\ 1)$.

D7b $f(x)=-4+3\cos(2x-\frac{1}{4}\pi)=-4+3\cos(2(x-\frac{1}{8}\pi))$ heeft
evenwichtsstand -4; amplitude 3; periode $\frac{2\pi}{2}=\pi$ en beginpunt $(\frac{1}{8}\pi,\ -4+3)=(\frac{1}{8}\pi,\ -1)$.
Hoogste punten zijn $(\frac{1}{8}\pi+k\cdot\pi,\ -4+3)=(\frac{1}{8}\pi+k\cdot\pi,\ -1)$.
Laagste punten zijn $(\frac{1}{8}\pi+\frac{1}{2}\cdot\pi+k\cdot\pi,\ 3-2)=(\frac{5}{8}\pi+k\cdot\pi,\ 1)$.

D8a $f(x)=\sin(\boxed{2x})-2\sin(x)\Rightarrow f'(x)=2\cos(2x)-2\cos(x)$.
$f(\pi)=\sin(2\pi)-2\sin(\pi)=0-2\cdot 0=0\Rightarrow A(\pi,0)$ en rc $=f'(\pi)=2\cos(2\pi)-2\cos(\pi)=2\cdot 1-2\cdot -1=2+2=4$.
$\left.\begin{array}{l}y=4x+b\\ \text{door } A(\pi,0)\end{array}\right\}\Rightarrow 0=4\pi+b\Rightarrow -4\pi=b$. Dus de raaklijn in $A(\pi,0)$ is $y=4x-4\pi$.

D8b $f'(x)=2\cos(2x)-2\cos(x)=0\Rightarrow 2\cos(2x)=2\cos(x)\Rightarrow\cos(2x)=\cos(x)\Rightarrow 2x=x+k\cdot 2\pi\vee 2x=-x+k\cdot 2\pi\Rightarrow$
$x=k\cdot 2\pi\vee 3x=k\cdot 2\pi\Rightarrow x=k\cdot 2\pi\vee x=k\cdot\frac{2}{3}\pi$. Dus (zie ook figuur 11.25) $x_B=\frac{2}{3}\pi$ en $x_C=\frac{4}{3}\pi$.
$y_B=f(\frac{2}{3}\pi)=\sin(\frac{4}{3}\pi)-2\sin(\frac{2}{3}\pi)=-\frac{1}{2}\sqrt{3}-2\cdot\frac{1}{2}\sqrt{3}=-\frac{1}{2}\sqrt{3}-\sqrt{3}=-1\frac{1}{2}\sqrt{3}\Rightarrow B(\frac{2}{3}\pi,\ -1\frac{1}{2}\sqrt{3})$.
$y_C=f(\frac{4}{3}\pi)=\sin(\frac{8}{3}\pi)-2\sin(\frac{4}{3}\pi)=\frac{1}{2}\sqrt{3}-2\cdot -\frac{1}{2}\sqrt{3}=\frac{1}{2}\sqrt{3}+\sqrt{3}=1\frac{1}{2}\sqrt{3}\Rightarrow C(\frac{4}{3}\pi,\ 1\frac{1}{2}\sqrt{3})$.

```
sin(4/3π)
   -.8660254038
Ans/√(3)
            -.5
sin(2/3π)
    .8660254038
```

D8c $f'(x)=2\cos(2x)-2\cos(x)=-2\Rightarrow\cos(2x)-\cos(x)=-1\Rightarrow 2\cos^2(x)-1-\cos(x)+1=0\Rightarrow 2\cos^2(x)-\cos(x)=0\Rightarrow$
$\cos(x)\cdot(2\cos(x)-1)=0\Rightarrow\cos(x)=0\vee\cos(x)=\frac{1}{2}\Rightarrow x=\frac{1}{2}\pi+k\cdot\pi\vee x=\frac{1}{3}\pi+k\cdot 2\pi\vee x=-\frac{1}{3}\pi+k\cdot 2\pi$.
x op $[0,2\pi]\Rightarrow x=\frac{1}{3}\pi\vee x=\frac{1}{2}\pi\vee x=1\frac{1}{2}\pi\vee x=1\frac{2}{3}\pi$.
$f(\frac{1}{3}\pi)=\sin(\frac{2}{3}\pi)-2\sin(\frac{1}{3}\pi)=\frac{1}{2}\sqrt{3}-2\cdot\frac{1}{2}\sqrt{3}=\frac{1}{2}\sqrt{3}-\sqrt{3}=-\frac{1}{2}\sqrt{3}\Rightarrow$ raakpunt $(\frac{1}{3}\pi,\ -\frac{1}{2}\sqrt{3})$.
$f(\frac{1}{2}\pi)=\sin(\pi)-2\sin(\frac{1}{2}\pi)=0-2\cdot 1=-2\Rightarrow$ raakpunt $(\frac{1}{2}\pi,\ -2)$.
$f(1\frac{1}{2}\pi)=\sin(3\pi)-2\sin(1\frac{1}{2}\pi)=0-2\cdot -1=2\Rightarrow$ raakpunt $(1\frac{1}{2}\pi,\ 2)$.
$f(1\frac{2}{3}\pi)=\sin(3\frac{1}{3}\pi)-2\sin(1\frac{2}{3}\pi)=-\frac{1}{2}\sqrt{3}-2\cdot -\frac{1}{2}\sqrt{3}=-\frac{1}{2}\sqrt{3}+\sqrt{3}=\frac{1}{2}\sqrt{3}\Rightarrow$ raakpunt $(1\frac{2}{3}\pi,\ \frac{1}{2}\sqrt{3})$.

D9a $f(x)=-\frac{1}{2}\sin(\boxed{2x+\frac{1}{2}\pi})\Rightarrow F(x)=-\frac{1}{2}\cdot\frac{1}{2}\cdot -\cos(2x+\frac{1}{2}\pi)+c=\frac{1}{4}\cos(2x+\frac{1}{2}\pi)+c$.

D9b $g(x)=3x^2+\cos(\boxed{\frac{1}{3}x})\Rightarrow G(x)=3\cdot\frac{1}{3}x^3+3\cdot\sin(\frac{1}{3}x)+c=x^3+3\sin(\frac{1}{3}x)+c$.

D9c $h(x)=x-2\sin^2(x)=x+\left(1-2\sin^2(x)\right)-1=x+\cos(\boxed{2x})-1\Rightarrow H(x)=\frac{1}{2}x^2+\frac{1}{2}\cdot\sin(2x)-x+c$.

D9d $k(x)=2+\tan^2(x)=1+\left(1+\tan^2(x)\right)\Rightarrow K(x)=x+\tan(x)+c$.

D10a $\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi}\left(\sin(\boxed{2x})+\cos(x)\right)dx=\left[-\frac{1}{2}\cos(2x)+\sin(x)\right]_{\frac{1}{6}\pi}^{\frac{1}{3}\pi}=-\frac{1}{2}\cos(\frac{2}{3}\pi)+\sin(\frac{1}{3}\pi)-\left(-\frac{1}{2}\cos(\frac{1}{3}\pi)+\sin(\frac{1}{6}\pi)\right)$

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cos(2/3π)
            -.5
sin(1/3π)
    .8660254038
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```
cos(1/3π)    .5
sin(1/6π)
             .5
```

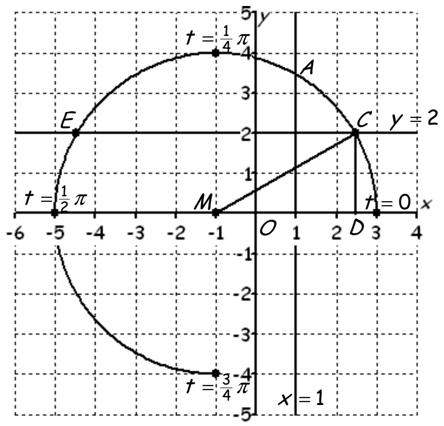
$=-\frac{1}{2}\cdot -\frac{1}{2}+\frac{1}{2}\sqrt{3}-\left(-\frac{1}{2}\cdot\frac{1}{2}+\frac{1}{2}\right)=\frac{1}{4}+\frac{1}{2}\sqrt{3}+\frac{1}{4}-\frac{1}{2}=\frac{1}{2}\sqrt{3}$.

D10b $\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi}\sin^2(x)\,dx=\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi}\left(\frac{1}{2}-\frac{1}{2}\cos(\boxed{2x})\right)dx=\left[\frac{1}{2}x-\frac{1}{2}\cdot\frac{1}{2}\sin(2x)\right]_{\frac{1}{6}\pi}^{\frac{1}{3}\pi}$
$\left[\cos(2A)=1-2\sin^2(A)\Rightarrow 2\sin^2(A)=1-\cos(2A)\Rightarrow\sin^2(A)=\frac{1}{2}-\frac{1}{2}\cos(2A)\right]$

```
sin(2/3π)
    .8660254038
sin(1/3π)
    .8660254038
```

$=\frac{1}{2}\cdot\frac{1}{3}\pi-\frac{1}{4}\sin(\frac{2}{3}\pi)-\left(\frac{1}{2}\cdot\frac{1}{6}\pi-\frac{1}{4}\sin(\frac{1}{3}\pi)\right)=\frac{1}{6}\pi-\frac{1}{4}\cdot\frac{1}{2}\sqrt{3}-\frac{1}{12}\pi+\frac{1}{4}\cdot\frac{1}{2}\sqrt{3}=\frac{1}{12}\pi$.

D11 $\int_{\frac{1}{2}\pi}^{\frac{3}{2}\pi}\pi\cdot(f(x))^2\,dx=\int_{\frac{1}{2}\pi}^{\frac{3}{2}\pi}\pi\cdot\cos^2(x)\,dx=\int_{\frac{1}{2}\pi}^{\frac{3}{2}\pi}\pi\cdot\left(\frac{1}{2}+\frac{1}{2}\cos(\boxed{2x})\right)dx=\left[\pi\cdot\left(\frac{1}{2}x+\frac{1}{2}\cdot\frac{1}{2}\sin(2x)\right)\right]_{\frac{1}{2}\pi}^{\frac{3}{2}\pi}$
$\left[\cos(2A)=2\cos^2(A)-1\Rightarrow 1+\cos(2A)=2\cos^2(A)\Rightarrow\frac{1}{2}+\frac{1}{2}\cos(2A)=\cos^2(A)\right]$
$=\pi\cdot\left(\frac{1}{2}\cdot\frac{3}{2}\pi+\frac{1}{4}\sin(3\pi)\right)-\pi\cdot\left(\frac{1}{2}\cdot\frac{1}{2}\pi+\frac{1}{4}\sin(\pi)\right)=\frac{3}{4}\pi^2+\frac{1}{4}\pi\cdot 0-\frac{1}{4}\pi^2-\frac{1}{4}\pi\cdot 0=\frac{1}{2}\pi^2$.



D12a De baan is een driekwartcirkel met middelpunt $(-1, 0)$ en straal 4.
t op $[0, \frac{3}{4}\pi] \Rightarrow 2t$ op $[0, 1\frac{1}{2}\pi] \Rightarrow$ driekwartcirkel.

D12b $x = 1 \Rightarrow -1 + 4\cos(2t) = 1 \Rightarrow 4\cos(2t) = 2 \Rightarrow \cos(2t) = \frac{1}{2} \Rightarrow$
$2t = \frac{1}{3}\pi + k \cdot 2\pi \vee 2t = -\frac{1}{3}\pi + k \cdot 2\pi \Rightarrow t = \frac{1}{6}\pi + k \cdot \pi \vee t = -\frac{1}{6}\pi + k \cdot \pi$.
Dus $t = \frac{1}{6}\pi$ en $y_A = 4\sin(2 \cdot \frac{1}{6}\pi) = 4\sin(\frac{1}{3}\pi) = 4 \cdot \frac{1}{2}\sqrt{3} \Rightarrow A(1, 2\sqrt{3})$.

D12c In ΔMDC geldt: $\sin\angle M = \frac{CD}{MC} = \frac{2}{4} = \frac{1}{2} \Rightarrow \angle M = \frac{1}{6}\pi$.
$\angle CME = \pi - 2 \cdot \frac{1}{6}\pi = \frac{2}{3}\pi$.
Dus de lengte boog CE is $\frac{\frac{2}{3}\pi}{2\pi} \cdot (\text{omtrek cirkel}) = \frac{\frac{2}{3}\pi}{2\pi} \cdot 2\pi \cdot 4 = \frac{2}{3}\pi \cdot 4 = \frac{8}{3}\pi$.

D13a De omlooptijd is 3 seconden $\Rightarrow \omega = \frac{2\pi}{3} = \frac{2}{3}\pi$.
De parametervoorstelling voor de baan van punt P is: $\begin{cases} x_P = 5 + 13\cos(\frac{2}{3}\pi(t-5)) \\ y_P = 12 + 13\sin(\frac{2}{3}\pi(t-5)) \end{cases}$ (met t in seconden).

D13b Punt Q met 1 seconde achterstand op P geeft als parametervoorstelling voor de baan van Q:
$\begin{cases} x_Q = 5 + 13\cos(\frac{2}{3}\pi(t-5-1)) \\ y_Q = 12 + 13\sin(\frac{2}{3}\pi(t-5-1)) \end{cases} \Rightarrow \begin{cases} x_Q = 5 + 13\cos(\frac{2}{3}\pi(t-6)) \\ y_Q = 12 + 13\sin(\frac{2}{3}\pi(t-6)) \end{cases}$ (met t in seconden).

D13c Punt R met een fasevoorsprong van $\frac{1}{4}$ op P geeft een voorsprong van $\frac{1}{4} \cdot 3 = \frac{3}{4}$ seconde.
De parametervoorstelling voor de baan van R is:
$\begin{cases} x_R = 5 + 13\cos(\frac{2}{3}\pi(t-5+\frac{3}{4})) \\ y_R = 12 + 13\sin(\frac{2}{3}\pi(t-5+\frac{3}{4})) \end{cases} \Rightarrow \begin{cases} x_R = 5 + 13\cos(\frac{2}{3}\pi(t-4\frac{1}{4})) \\ y_R = 12 + 13\sin(\frac{2}{3}\pi(t-4\frac{1}{4})) \end{cases}$ (met t in seconden).

Gemengde opgaven 10. Goniometrie en beweging

G25a $\frac{2\sin(x)\cdot\cos(x)}{1-2\sin^2(x)} = \frac{\sin(2x)}{\cos(2x)} = \tan(2x).$

G25b $\cos^4(x) - \sin^4(x) = \left(\cos^2(x) + \sin^2(x)\right)\cdot\left(\cos^2(x) - \sin^2(x)\right) = 1\cdot\cos(2x) = \cos(2x).$

G25c $\frac{\sin(2x)}{1+\cos(2x)} = \frac{2\sin(x)\cdot\cos(x)}{2\cos^2(x)} = \frac{\sin(x)}{\cos(x)} = \tan(x).$

G25d $\cos(x-y)\cdot\cos(y) - \sin(x-y)\cdot\sin(y) = \cos(x-y+y) = \cos(x).$

G26a
$\sin(x)\cdot\cos(x) = \frac{1}{4}$
$2\sin(x)\cdot\cos(x) = \frac{1}{2}$
$\sin(2x) = \frac{1}{2}$
$2x = \frac{1}{6}\pi + k\cdot 2\pi \vee 2x = \frac{5}{6}\pi + k\cdot 2\pi$
$x = \frac{1}{12}\pi + k\cdot\pi \vee x = \frac{5}{12}\pi + k\cdot\pi.$

G26c
$\cos(x + \frac{1}{3}\pi) = -\sin(x)$
$\sin(x + \frac{1}{3}\pi + \frac{1}{2}\pi) = \sin(x+\pi)$
$x + \frac{5}{6}\pi = x + \pi + k\cdot 2\pi \vee x + \frac{5}{6}\pi = \pi - x - \pi + k\cdot 2\pi$
geen oplossing $\vee\ 2x = -\frac{5}{6}\pi + k\cdot 2\pi$
$x = -\frac{5}{12}\pi + k\cdot\pi.$

G26b
$\cos(x - \frac{1}{3}\pi) = \sin(2x)$
$\cos(x - \frac{1}{3}\pi) = \cos(2x - \frac{1}{2}\pi)$
$x - \frac{1}{3}\pi = 2x - \frac{1}{2}\pi + k\cdot 2\pi \vee x - \frac{1}{3}\pi = -2x + \frac{1}{2}\pi + k\cdot 2\pi$
$-x = -\frac{1}{6}\pi + k\cdot 2\pi \vee 3x = \frac{5}{6}\pi + k\cdot 2\pi$
$x = \frac{1}{6}\pi + k\cdot 2\pi \vee x = \frac{5}{18}\pi + k\cdot\frac{2}{3}\pi.$

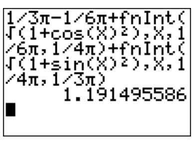
G26d
$\cos(2x) - \sin^2(x) = \frac{1}{4}$
$\cos(2x) + \frac{1}{2}\cos(2x) - \frac{1}{2} = \frac{1}{4}$
$1\frac{1}{2}\cos(2x) = \frac{3}{4} \Rightarrow \cos(2x) = \frac{1}{2}$
$2x = \frac{1}{3}\pi + k\cdot 2\pi \vee 2x = -\frac{1}{3}\pi + k\cdot 2\pi$
$x = \frac{1}{6}\pi + k\cdot\pi \vee x = -\frac{1}{6}\pi + k\cdot\pi.$

G27a
$f(x) = \frac{1}{2}$, $\sin(x) = \frac{1}{2}$, $x = \frac{1}{6}\pi.$
$f(x) = g(x)$, $\sin(x) = \cos(x)$, $x = \frac{1}{4}\pi.$
$g(x) = \frac{1}{2}$, $\cos(x) = \frac{1}{2}$, $x = \frac{1}{3}\pi.$

$$O(V) = \int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi}\left(\sin(x) - \tfrac{1}{2}\right)dx + \int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi}\left(\cos(x) - \tfrac{1}{2}\right)dx = \left[-\cos(x) - \tfrac{1}{2}x\right]_{\frac{1}{6}\pi}^{\frac{1}{4}\pi} + \left[\sin(x) - \tfrac{1}{2}x\right]_{\frac{1}{4}\pi}^{\frac{1}{3}\pi}$$
$$= -\cos(\tfrac{1}{4}\pi) - \tfrac{1}{2}\cdot\tfrac{1}{4}\pi - \left(-\cos(\tfrac{1}{6}\pi) - \tfrac{1}{2}\cdot\tfrac{1}{6}\pi\right) + \sin(\tfrac{1}{3}\pi) - \tfrac{1}{2}\cdot\tfrac{1}{3}\pi - \left(\sin(\tfrac{1}{4}\pi) - \tfrac{1}{2}\cdot\tfrac{1}{4}\pi\right)$$
$$= -\tfrac{1}{2}\sqrt{2} - \tfrac{1}{8}\pi + \tfrac{1}{2}\sqrt{3} + \tfrac{1}{12}\pi + \tfrac{1}{2}\sqrt{3} - \tfrac{1}{6}\pi - \tfrac{1}{2}\sqrt{2} + \tfrac{1}{8}\pi = -\sqrt{2} + \sqrt{3} - \tfrac{1}{12}\pi.$$

G27b
$$I(L) = \int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi}\pi\cdot\sin^2(x)\,dx + \int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi}\pi\cdot\cos^2(x)\,dx - \pi\cdot(\tfrac{1}{2})^2\cdot(\tfrac{1}{3}\pi - \tfrac{1}{6}\pi)$$
$$= \int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi}\pi\cdot\left(\tfrac{1}{2} - \tfrac{1}{2}\cos(\boxed{2x})\right)dx + \int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi}\pi\cdot\left(\tfrac{1}{2} + \tfrac{1}{2}\cos(\boxed{2x})\right)dx - \tfrac{1}{24}\pi^2$$
$$= \left[\pi\cdot\left(\tfrac{1}{2}x - \tfrac{1}{4}\sin(2x)\right)\right]_{\frac{1}{6}\pi}^{\frac{1}{4}\pi} + \left[\pi\cdot\left(\tfrac{1}{2}x + \tfrac{1}{4}\sin(2x)\right)\right]_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} - \tfrac{1}{24}\pi^2$$
$$= \pi\cdot\left(\tfrac{1}{2}\cdot\tfrac{1}{4}\pi - \tfrac{1}{4}\cdot\sin(\tfrac{1}{2}\pi)\right) - \pi\cdot\left(\tfrac{1}{2}\cdot\tfrac{1}{6}\pi - \tfrac{1}{4}\cdot\sin(\tfrac{1}{3}\pi)\right) + \pi\cdot\left(\tfrac{1}{2}\cdot\tfrac{1}{3}\pi + \tfrac{1}{4}\cdot\sin(\tfrac{2}{3}\pi)\right) - \pi\cdot\left(\tfrac{1}{2}\cdot\tfrac{1}{4}\pi + \tfrac{1}{4}\cdot\sin(\tfrac{1}{2}\pi)\right) - \tfrac{1}{24}\pi^2$$
$$= \tfrac{1}{8}\pi^2 - \tfrac{1}{4}\pi\cdot 1 - \tfrac{1}{12}\pi^2 + \tfrac{1}{8}\pi\sqrt{3} + \tfrac{1}{6}\pi^2 + \tfrac{1}{8}\pi\sqrt{3} - \tfrac{1}{8}\pi^2 - \tfrac{1}{4}\pi\cdot 1 - \tfrac{1}{24}\pi^2 = \tfrac{1}{24}\pi^2 - \tfrac{1}{2}\pi + \tfrac{1}{4}\pi\sqrt{3}.$$

G27c
$$\text{omtrek} = \tfrac{1}{3}\pi - \tfrac{1}{6}\pi + \int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi}\sqrt{1 + \left(f'(x)\right)^2}\,dx + \int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi}\sqrt{1 + \left(g'(x)\right)^2}\,dx$$
$$= \tfrac{1}{3}\pi - \tfrac{1}{6}\pi + \int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi}\sqrt{1 + \cos^2(x)}\,dx + \int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi}\sqrt{1 + \sin^2(x)}\,dx \text{ (fnInt)} \approx 1{,}19.$$



G28a $f(x) = 2\left(\boxed{\cos(x)}\right)^2 + \sin(\boxed{2x}) \Rightarrow f'(x) = 4\cos(x)\cdot -\sin(x) + 2\cos(2x) = -2\sin(2x) + 2\cos(2x).$

$f'(x) = 0 \Rightarrow -2\sin(2x) + 2\cos(2x) = 0 \Rightarrow 2\cos(2x) = 2\sin(2x) \Rightarrow \cos(2x) = \sin(2x) \Rightarrow \cos(2x) = \cos(2x - \frac{1}{2}\pi) \Rightarrow$

$2x = 2x - \frac{1}{2}\pi + k\cdot 2\pi$ (geen oplossing) $\vee\ 2x = -2x + \frac{1}{2}\pi + k\cdot 2\pi \Rightarrow 4x = \frac{1}{2}\pi + k\cdot 2\pi \Rightarrow x = \frac{1}{8}\pi + k\cdot\frac{1}{2}\pi.$

$f(x) = 2\cos^2(x) + \sin(2x) = 2\cos^2(x) - 1 + 1 + \sin(2x) = \cos(2x) + 1 + \sin(2x).$

absoluut maximum (zie fig G.13): $f(\frac{1}{8}\pi) = \cos(\frac{1}{4}\pi) + 1 + \sin(\frac{1}{4}\pi) = \frac{1}{2}\sqrt{2} + 1 + \frac{1}{2}\sqrt{2} = 1 + \sqrt{2}$

absoluut minimum (zie fig G.13): $f(\frac{5}{8}\pi) = \cos(\frac{5}{4}\pi) + 1 + \sin(\frac{5}{4}\pi) = -\frac{1}{2}\sqrt{2} + 1 - \frac{1}{2}\sqrt{2} = 1 - \sqrt{2}$

$\Rightarrow B_f = [1 - \sqrt{2}, 1 + \sqrt{2}].$

G28b 💻 $f(x)=g(x)\Rightarrow 2\cos^2(x)+\sin(2x)=2\sin(2x)$

$2\cos^2(x)=\sin(2x)$

$2\cos^2(x)=2\sin(x)\cos(x)$

$2\cos^2(x)-2\sin(x)\cos(x)=0$

$2\cos(x)\left(\cos(x)-\sin(x)\right)=0$

$\cos(x)=0\vee\cos(x)=\sin(x)$

$x=\frac12\pi+k\cdot\pi\vee\cos(x)=\cos(x-\frac12\pi)$

$x=\frac12\pi+k\cdot\pi\vee x=x-\frac12\pi+k\cdot 2\pi\vee x=-x+\frac12\pi+k\cdot2\pi$

$x=\frac12\pi+k\cdot\pi\vee$ geen oplossing $\vee 2x=+\frac12\pi+k\cdot2\pi$

$x=\frac12\pi+k\cdot\pi\vee x=\frac14\pi+k\cdot\pi.$

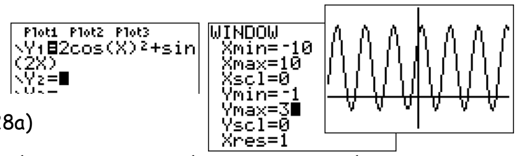
x op $[0,\pi]\Rightarrow x=\frac12\pi\vee x=\frac14\pi.$

$$O(V)=\int_{\frac14\pi}^{\frac12\pi}\left(2\sin(2x)-\left(2\cos^2(x)+\sin(2x)\right)\right)\,dx$$
$$=\int_{\frac14\pi}^{\frac12\pi}\left(\sin(2x)-2\cos^2(x)\right)dx$$
$$=\int_{\frac14\pi}^{\frac12\pi}\left(\sin(\boxed{2x})-\cos(\boxed{2x})-1\right)dx$$
$$=\left[-\tfrac12\cos(2x)-\tfrac12\sin(2x)-x\right]_{\frac14\pi}^{\frac12\pi}$$
$$=-\tfrac12\cos(\pi)-\tfrac12\sin(\pi)-\tfrac12\pi+\tfrac12\cos(\tfrac12\pi)+\tfrac12\sin(\tfrac12\pi)+\tfrac14\pi$$
$$=\tfrac12-0-\tfrac12\pi+0+\tfrac12+\tfrac14\pi=1-\tfrac14\pi.$$

G28c 💻 C het midden van AB als $g(p)=\frac12 f(p)\Rightarrow 2\sin(2p)=\cos^2(p)+\frac12\sin(2p)\Rightarrow$

$1\frac12\sin(2p)=\cos^2(p)\Rightarrow 3\sin(p)\cos(p)=\cos(p)\cos(p)\Rightarrow$

$\cos(p)=0$(voldoet niet) $\vee 3\sin(p)=\cos(p)\Rightarrow 3\frac{\sin(p)}{\cos(p)}=1\Rightarrow 3\tan(p)=1\Rightarrow\tan(p)=\frac13.$



G29a 💻 De grafiek van f (dezelfde als in G28) is vermoedelijk lijnsymmetrisch in de verticale lijn door de eerste top rechts van de y-as.

Vermoedelijk lijnsymmetrisch in de lijn $x=\frac18\pi$. (zie de berekening in G28a)

$f(\frac18\pi+p)=2\cos^2(\frac18\pi+p)+\sin(\frac14\pi+2p)=2\cos^2(\frac18\pi+p)-1+1+\sin(\frac14\pi+2p)=\cos(\frac14\pi+2p)+\sin(\frac14\pi+2p)+1$

$=\cos(\frac14\pi)\cos(2p)-\sin(\frac14\pi)\sin(2p)+\sin(\frac14\pi)\cos(2p)+\cos(\frac14\pi)\sin(2p)+1$

$=\frac12\sqrt2\cdot\cos(2p)-\frac12\sqrt2\cdot\sin(2p)+\frac12\sqrt2\cdot\cos(2p)+\frac12\sqrt2\cdot\sin(2p)+1=\sqrt2\cos(2p)+1.$

$f(\frac18\pi-p)=2\cos^2(\frac18\pi-p)+\sin(\frac14\pi-2p)=2\cos^2(\frac18\pi-p)-1+1+\sin(\frac14\pi-2p)=\cos(\frac14\pi-2p)+\sin(\frac14\pi-2p)+1$

$=\cos(\frac14\pi)\cos(2p)+\sin(\frac14\pi)\sin(2p)+\sin(\frac14\pi)\cos(2p)-\cos(\frac14\pi)\sin(2p)+1$

$=\frac12\sqrt2\cdot\cos(2p)+\frac12\sqrt2\cdot\sin(2p)+\frac12\sqrt2\cdot\cos(2p)-\frac12\sqrt2\cdot\sin(2p)+1=\sqrt2\cos(2p)+1.$

$f(\frac18\pi+p)=f(\frac18\pi-p)$ (voor elke p) $\Rightarrow$ de grafiek van f is symmetrisch in de lijn $x=\frac18\pi$.

G29b 💻 Vermoedelijk puntsymmetrisch in $A(-\frac18\pi,1)$. (A precies midden tussen de toppen bij $x=\frac18\pi$ en $x=\frac18\pi-\frac12\pi$ zie G28a)

$f(-\frac18\pi+p)=2\cos^2(-\frac18\pi+p)+\sin(-\frac14\pi+2p)=\cos(-\frac14\pi+2p)+\sin(-\frac14\pi+2p)+1$

$=\cos(-\frac14\pi)\cos(2p)-\sin(-\frac14\pi)\sin(2p)+\sin(-\frac14\pi)\cos(2p)+\cos(-\frac14\pi)\sin(2p)+1$

$=\frac12\sqrt2\cdot\cos(2p)+\frac12\sqrt2\cdot\sin(2p)-\frac12\sqrt2\cdot\cos(2p)+\frac12\sqrt2\cdot\sin(2p)+1=\sqrt2\sin(2p)+1.$

$f(-\frac18\pi-p)=2\cos^2(-\frac18\pi-p)+\sin(-\frac14\pi-2p)=\cos(-\frac14\pi-2p)+\sin(-\frac14\pi-2p)+1$

$=\cos(-\frac14\pi)\cos(2p)+\sin(-\frac14\pi)\sin(2p)+\sin(-\frac14\pi)\cos(2p)-\cos(-\frac14\pi)\sin(2p)+1$

$=\frac12\sqrt2\cdot\cos(2p)-\frac12\sqrt2\cdot\sin(2p)-\frac12\sqrt2\cdot\cos(2p)-\frac12\sqrt2\cdot\sin(2p)+1=-\sqrt2\sin(2p)+1.$

$f(-\frac18\pi+p)+f(-\frac18\pi-p)=\sqrt2\sin(2p)+1+-\sqrt2\sin(2p)+1=2\Rightarrow f$ is symmetrisch in $A(-\frac18\pi,1)$.

G30a 💻 $f(x)=0\Rightarrow 2\sin^2(x)+\sin(x)=0\Rightarrow\sin(x)\cdot(2\sin(x)+1)=0\Rightarrow\sin(x)=0\vee\sin(x)=-\frac12\Rightarrow$

$x=k\cdot\pi\vee x=-\frac16\pi+k\cdot2\pi\vee x=1\frac16\pi+k\cdot2\pi.$ x op $[0,2\pi]\Rightarrow$ nulp.: $x=0\vee x=\pi\vee x=2\pi\vee x=1\frac56\pi\vee x=1\frac16\pi.$

G30b 💻 $O(V)=\int_0^{\pi}\left(2\sin^2(x)+\sin(x)\right)dx+\int_0^{\pi}\left(1-\cos(\boxed{2x})+\sin(x)\right)dx=\left[x-\frac12\sin(2x)-\cos(x)\right]_0^{\pi}$

$=\pi-\frac12\sin(2\pi)-\cos(\pi)-\left(0-\frac12\sin(0)-\cos(0)\right)=\pi-0+1-(0-0-1)=\pi+2.$

G30c 💻 $f(x)=2\sin^2(x)+\sin(x)=2\left(\boxed{\sin(x)}\right)^2+\sin(x)\Rightarrow f'(x)=4\sin(x)\cos(x)+\cos(x)$

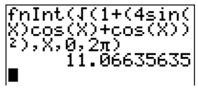
$f'(x)=0\Rightarrow 4\sin(x)\cos(x)+\cos(x)=0\Rightarrow\cos(x)=0\vee 4\sin(x)+1=0\Rightarrow x=\frac12\pi+k\cdot\pi\vee\sin(x)=-\frac14.$

$x=1\frac12\pi\Rightarrow f(x)=f(1\frac12\pi)=2\sin^2(1\frac12\pi)+\sin(1\frac12\pi)=2\cdot(-1)^2+-1=2-1=1.$

$\sin(x)=-\frac14\Rightarrow f(x)=2\cdot(-\frac14)^2+-\frac14=\frac{2}{16}-\frac14=-\frac{2}{16}=-\frac18.$

$f(x)=p$ heeft precies vier oplossingen (zie figuur G.14 en de berekening hierboven) voor $-\frac18<p<0\vee 0<p<1.$

G30d $L(\text{grafiek van } f)=\int_0^{2\pi}\sqrt{1+(f'(x))^2}\,\mathrm{d}x=\int_0^{2\pi}\sqrt{1+(4\sin(x)\cos(x)+\cos(x))^2}\,\mathrm{d}x$ (fnInt) $\approx 11{,}07.$



G31a $g(x)=2\sin(x)+\cos(\boxed{2x})\Rightarrow g'(x)=2\cos(x)-2\sin(2x).$

$g'(x)=0\Rightarrow\cos(x)=\sin(2x)\Rightarrow\cos(x)=\cos(2x-\tfrac{1}{2}\pi)$

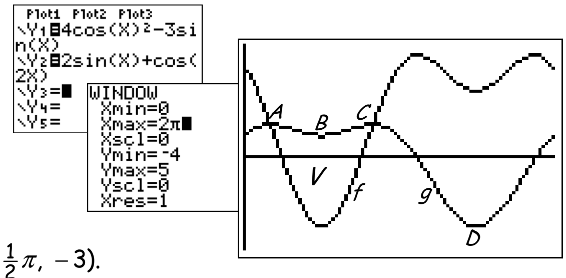
$x=2x-\tfrac{1}{2}\pi+k\cdot 2\pi\vee x=-2x+\tfrac{1}{2}\pi+k\cdot 2\pi$

$-x=-\tfrac{1}{2}\pi+k\cdot 2\pi\vee 3x=\tfrac{1}{2}\pi+k\cdot 2\pi$

$x=\tfrac{1}{2}\pi+k\cdot 2\pi\vee x=\tfrac{1}{6}\pi+k\cdot\tfrac{2}{3}\pi.$

x op $[0,2\pi]\Rightarrow x=\tfrac{1}{2}\pi\vee x=\tfrac{1}{6}\pi\vee x=\tfrac{5}{6}\pi\vee x=1\tfrac{1}{2}\pi.$

Dit geeft toppen: $A(\tfrac{1}{6}\pi,1\tfrac{1}{2})$, $B(\tfrac{1}{2}\pi,1)$, $C(\tfrac{5}{6}\pi,1\tfrac{1}{2})$ en $D(1\tfrac{1}{2}\pi,-3)$.



$f(\tfrac{1}{6}\pi)=4\cos^2(\tfrac{1}{6}\pi)-3\sin(\tfrac{1}{6}\pi)=4\cdot(\tfrac{1}{2}\sqrt{3})^2-3\cdot\tfrac{1}{2}=4\cdot\tfrac{1}{4}\cdot 3-1\tfrac{1}{2}=3-1\tfrac{1}{2}=1\tfrac{1}{2}$

$f(\tfrac{5}{6}\pi)=4\cos^2(\tfrac{5}{6}\pi)-3\sin(\tfrac{5}{6}\pi)=4\cdot(-\tfrac{1}{2}\sqrt{3})^2-3\cdot\tfrac{1}{2}=4\cdot\tfrac{1}{4}\cdot 3-1\tfrac{1}{2}=1\tfrac{1}{2}$

$\Rightarrow A$ en C liggen op de grafiek van f.

G31b $O(V)=\int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi}\left(2\sin(x)+\cos(2x)-\left(4\cos^2(x)-3\sin(x)\right)\right)\mathrm{d}x=\int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi}\left(2\sin(x)+\cos(2x)-4\cos^2(x)+3\sin(x)\right)\mathrm{d}x$

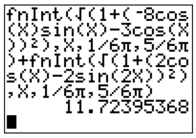
$=\int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi}\left(5\sin(x)+\cos(2x)-2\cdot(\cos(2x)+1)\right)\mathrm{d}x=\int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi}\left(5\sin(x)-\cos(\boxed{2x})-2\right)\mathrm{d}x=\left[-5\cos(x)-\tfrac{1}{2}\sin(2x)-2x\right]_{\frac{1}{6}\pi}^{\frac{5}{6}\pi}$

$=-5\cos(\tfrac{5}{6}\pi)-\tfrac{1}{2}\sin(\tfrac{5}{3}\pi)-\tfrac{5}{3}\pi-\left(-5\cos(\tfrac{1}{6}\pi)-\tfrac{1}{2}\sin(\tfrac{1}{3}\pi)-\tfrac{1}{3}\pi\right)$

$=-5\cdot-\tfrac{1}{2}\sqrt{3}-\tfrac{1}{2}\cdot-\tfrac{1}{2}\sqrt{3}-\tfrac{5}{3}\pi-\left(-5\cdot\tfrac{1}{2}\sqrt{3}-\tfrac{1}{2}\cdot\tfrac{1}{2}\sqrt{3}-\tfrac{1}{3}\pi\right)=2\tfrac{1}{2}\sqrt{3}+\tfrac{1}{4}\sqrt{3}-\tfrac{5}{3}\pi+2\tfrac{1}{2}\sqrt{3}+\tfrac{1}{4}\sqrt{3}+\tfrac{1}{3}\pi=5\tfrac{1}{2}\sqrt{3}-\tfrac{4}{3}\pi.$

G31c $\text{omtrek}(V)=\int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi}\sqrt{1+(f'(x))^2}\,\mathrm{d}x+\int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi}\sqrt{1+(g'(x))^2}\,\mathrm{d}x$

$=\int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi}\sqrt{1+(-8\cos(x)\sin(x)-3\cos(x)^2}\,\mathrm{d}x+\int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi}\sqrt{1+(2\cos(x)-2\sin(2x))^2}\,\mathrm{d}x$ (fnInt) $\approx 11{,}72.$

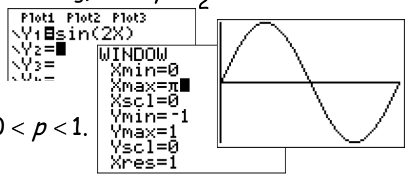


G32a $f_p(x)=\sin^2(x)+p\cos(2x)=\sin^2(x)+p\cdot\left(1-2\sin^2(x)\right)=p$ (de andere termen vallen weg) voor $p=\tfrac{1}{2}$.

G32b $f_p(x)=\sin^2(x)+p\cos(2x)=\left(\boxed{\sin(x)}\right)^2+p\cos(2x)\Rightarrow$

$f_p'(x)=2\sin(x)\cos(x)-2p\sin(2x)=\sin(2x)-2p\sin(2x)=(1-2p)\cdot\sin(2x).$

$f_p'(x)=(1-2p)\cdot\sin(2x)\neq 1\ \Rightarrow -1<1-2p<1\Rightarrow -2<-2p<0\Rightarrow 1>p>0\Rightarrow 0<p<1.$



G32c $\int_0^a f_p(x)\,\mathrm{d}x=\int_0^a\left(\sin^2(x)+p\cos(2x)\right)\mathrm{d}x=\int_0^a\left(\tfrac{1}{2}-\tfrac{1}{2}\cos(2x)+p\cos(2x)\right)\mathrm{d}x=\int_0^a\left(\tfrac{1}{2}+(p-\tfrac{1}{2})\cos(\boxed{2x})\right)\mathrm{d}x$

$=\left[\tfrac{1}{2}x+\tfrac{1}{2}(p-\tfrac{1}{2})\sin(2x)\right]_0^a=\tfrac{1}{2}a+\tfrac{1}{2}(p-\tfrac{1}{2})\sin(2a)-\left(\tfrac{1}{2}\cdot 0+\tfrac{1}{2}(p-\tfrac{1}{2})\sin(0)\right)=\tfrac{1}{2}a+\tfrac{1}{2}(p-\tfrac{1}{2})\sin(2a).$

Onafhankelijk van p als $\sin(2a)=0\Rightarrow 2a=k\cdot\pi\Rightarrow a=k\cdot\tfrac{1}{2}\pi.$ Gegeven: a op $[0,\pi]\Rightarrow a=0\vee a=\tfrac{1}{2}\pi\vee a=\pi.$

G33a $f(x)=\dfrac{3\cos(x)}{2+\sin(x)}\Rightarrow f'(x)=\dfrac{(2+\sin(x))\cdot-3\sin(x)-3\cos(x)\cdot\cos(x)}{(2+\sin(x))^2}=\dfrac{-6\sin(x)-3\sin^2(x)-3\cos^2(x)}{(2+\sin(x))^2}=\dfrac{-6\sin(x)-3}{(2+\sin(x))^2}.$

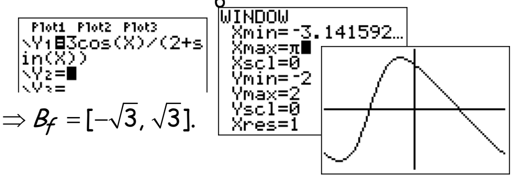
$f'(x)=0$ (teller = 0) $\Rightarrow -6\sin(x)-3=0\Rightarrow\sin(x)=-\tfrac{1}{2}\Rightarrow x=1\tfrac{1}{6}\pi+k\cdot 2\pi\vee x=\pi-1\tfrac{1}{6}\pi+k\cdot 2\pi.$

Gegeven: x op $[-\pi,\pi]\Rightarrow x=-\tfrac{5}{6}\pi\vee x=-\tfrac{1}{6}\pi.$

minimum (zie plot): $f(-\tfrac{5}{6}\pi)=\dfrac{3\cos(-\frac{5}{6}\pi)}{2+\sin(-\frac{5}{6}\pi)}=\dfrac{3\cdot-\frac{1}{2}\sqrt{3}}{2-\frac{1}{2}}=\dfrac{-1\frac{1}{2}\sqrt{3}}{1\frac{1}{2}}=-\sqrt{3}$

maximum (zie plot): $f(-\tfrac{1}{6}\pi)=\dfrac{3\cos(-\frac{1}{6}\pi)}{2+\sin(-\frac{1}{6}\pi)}=\dfrac{3\cdot\frac{1}{2}\sqrt{3}}{2-\frac{1}{2}}=\dfrac{1\frac{1}{2}\sqrt{3}}{1\frac{1}{2}}=\sqrt{3}$

$\Rightarrow B_f=[-\sqrt{3},\sqrt{3}].$

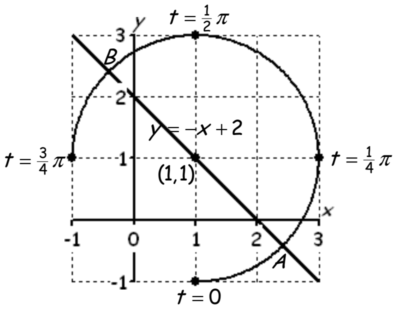


G33b $f(x)\cdot f(-x)=\tfrac{9}{7}\Rightarrow\dfrac{3\cos(x)}{2+\sin(x)}\cdot\dfrac{3\cos(-x)}{2+\sin(-x)}=\tfrac{9}{7}\Rightarrow\dfrac{3\cos(x)}{2+\sin(x)}\cdot\dfrac{3\cos(x)}{2-\sin(x)}=\tfrac{9}{7}\Rightarrow\dfrac{\cos^2(x)}{4-\sin^2(x)}=\tfrac{1}{7}\Rightarrow$

$7\cos^2(x)=4-\left(1-\cos^2(x)\right)\Rightarrow 6\cos^2(x)=3\Rightarrow\cos^2(x)=\tfrac{1}{2}\Rightarrow\cos(x)=\pm\sqrt{\tfrac{1}{2}}=\pm\sqrt{\tfrac{1}{2}\cdot\tfrac{2}{2}}=\pm\tfrac{1}{2}\cdot\sqrt{2}$

$x=\tfrac{1}{4}\pi+k\cdot\tfrac{1}{2}\pi.$ Gegeven: x op $[-\pi,\pi]\Rightarrow x=-\tfrac{3}{4}\pi\vee x=-\tfrac{1}{4}\pi\vee x=\tfrac{1}{4}\pi\vee x=\tfrac{3}{4}\pi.$

G34a t op $[0, \frac{3}{4}\pi] \Rightarrow 2t$ op $[0, 1\frac{1}{2}\pi]$. De baan is driekwartcirkel met middelpunt (1, 1) en straal 2.



G34b $y = -x + 2 \Rightarrow 1 + 2\sin(2t - \frac{1}{2}\pi) = -1 - 2\cos(2t - \frac{1}{2}\pi) + 2$

$\sin(2t - \frac{1}{2}\pi) = -\cos(2t - \frac{1}{2}\pi)$

$\cos(2t - \frac{1}{2}\pi - \frac{1}{2}\pi) = \cos(2t - \frac{1}{2}\pi + \pi)$

$2t - \pi = 2t + \frac{1}{2}\pi + k \cdot 2\pi \vee 2t - \pi = -2t - \frac{1}{2}\pi + k \cdot 2\pi$

geen oplossing $\quad 4t = \frac{1}{2}\pi + k \cdot 2\pi$

$t = \frac{1}{8}\pi + k \cdot \frac{1}{2}\pi.$

t op $[0, \frac{3}{4}\pi] \Rightarrow t = \frac{1}{8}\pi \vee t = \frac{5}{8}\pi.$

$t = \frac{1}{8}\pi \Rightarrow x_P = 1 + 2\cos(-\frac{1}{4}\pi) = 1 + 2 \cdot \frac{1}{2}\sqrt{2} = 1 + \sqrt{2}$ en $y_P = 1 + 2\sin(-\frac{1}{4}\pi) = 1 + 2 \cdot -\frac{1}{2}\sqrt{2} = 1 - \sqrt{2} \Rightarrow A(1 + \sqrt{2}, 1 - \sqrt{2}).$

$t = \frac{5}{8}\pi \Rightarrow x_P = 1 + 2\cos(\frac{3}{4}\pi) = 1 + 2 \cdot -\frac{1}{2}\sqrt{2} = 1 - \sqrt{2}$ en $y_P = 1 + 2\sin(\frac{3}{4}\pi) = 1 + 2 \cdot \frac{1}{2}\sqrt{2} = 1 + \sqrt{2} \Rightarrow B(1 - \sqrt{2}, 1 + \sqrt{2}).$

G34c $x = 0$ (y-as)

$1 + 2\cos(2t - \frac{1}{2}\pi) = 0$

$2\cos(2t - \frac{1}{2}\pi) = -1$

$\cos(2t - \frac{1}{2}\pi) = -\frac{1}{2}$

$2t - \frac{1}{2}\pi = \frac{2}{3}\pi + k \cdot 2\pi \vee 2t - \frac{1}{2}\pi = -\frac{2}{3}\pi + k \cdot 2\pi$

$2t = \frac{7}{6}\pi + k \cdot 2\pi \vee 2t = -\frac{1}{6}\pi + k \cdot 2\pi$

$t = \frac{7}{12}\pi + k \cdot \pi \vee t = -\frac{1}{12}\pi + k \cdot \pi$

$x > 0$ (zie de baan van P) $\Rightarrow 0 \le t < \frac{7}{12}\pi$

$y = 0$ (x-as)

$1 + 2\sin(2t - \frac{1}{2}\pi) = 0$

$2\sin(2t - \frac{1}{2}\pi) = -1$

$\sin(2t - \frac{1}{2}\pi) = -\frac{1}{2}$

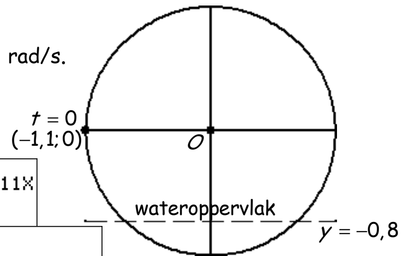
$2t - \frac{1}{2}\pi = -\frac{1}{6}\pi + k \cdot 2\pi \vee 2t - \frac{1}{2}\pi = 1\frac{1}{6}\pi + k \cdot 2\pi$

$2t = \frac{1}{3}\pi + k \cdot 2\pi \vee 2t = 1\frac{2}{3}\pi + k \cdot 2\pi$

$t = \frac{1}{6}\pi + k \cdot \pi \vee t = \frac{5}{6}\pi + k \cdot \pi$

$y > 0$ (zie de baan van P) $\Rightarrow \frac{1}{6}\pi < t \le \frac{3}{4}\pi$

Uit bovenstaande regel volgt dan: $x > 0$ en tevens $y > 0$ (zie de baan van P) $\Rightarrow \frac{1}{6}\pi < t < \frac{7}{12}\pi.$

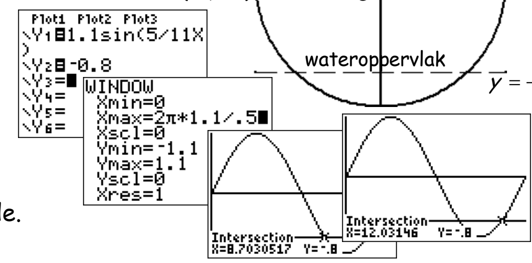


G35a $v = 0{,}5$ m/s $\Rightarrow$ per seconde $\frac{0{,}5}{2\pi \cdot 1{,}1}$ gedeelte van de cirkel $\Rightarrow \frac{0{,}5}{2\pi \cdot 1{,}1} \cdot 2\pi$ rad/s $= \frac{5}{11}$ rad/s.

G35b $\begin{cases} x = 1{,}1\cos(\frac{5}{11}t + \pi) \\ y = 1{,}1\sin(\frac{5}{11}t + \pi) \end{cases}$ (t in seconden en x en y in meters).

G35c De volgende koker loopt $\frac{1}{6}$ cirkel $= \frac{1}{6} \cdot 2\pi = \frac{1}{3}\pi$ radialen achter.

$\begin{cases} x = 1{,}1\cos(\frac{5}{11}t + \frac{2}{3}\pi) \\ y = 1{,}1\sin(\frac{5}{11}t + \frac{2}{3}\pi) \end{cases}$ (t in seconden en x en y in meters).



G35d $y = -0{,}8 \Rightarrow 1{,}1\sin(\frac{5}{11}t) = -0{,}8$ (intersect) $\Rightarrow t \approx 8{,}70 \vee t \approx 12{,}03.$

$y < -0{,}8$ (zie plot) $\Rightarrow 8{,}70 < t < 12{,}03$. Dus gedurende 3,3 seconde.

G36a $f_2(x) = 1 + \sin^2(x) + \cos(2x)$

$= 1 + \frac{1}{2} - \frac{1}{2}\cos(2x) + \cos(2x)$

$= 1\frac{1}{2} + \frac{1}{2}\cos(2x)$

$= 1\frac{1}{2} + \frac{1}{2}\sin(2x + \frac{1}{2}\pi)$

$= 1\frac{1}{2} + \frac{1}{2}\sin(2(x + \frac{1}{4}\pi)).$

Dit geeft $a = 1\frac{1}{2},\ b = \frac{1}{2},\ c = 2$ en $d = -\frac{1}{4}\pi.$

G36b $f_n(\frac{1}{6}\pi) = \frac{1}{4} \Rightarrow 1 + \sin^2(\frac{1}{6}\pi) + \cos(n \cdot \frac{1}{6}\pi) = \frac{1}{4}$

$1 + \left(\frac{1}{2}\right)^2 + \cos(n \cdot \frac{1}{6}\pi) = \frac{1}{4}$

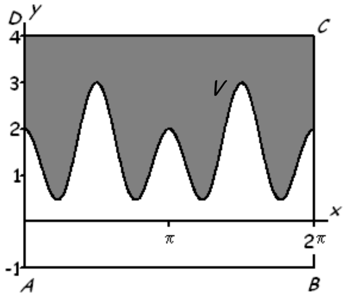
$\cos(n \cdot \frac{1}{6}\pi) = -1$

$n \cdot \frac{1}{6}\pi = \pi + k \cdot 2\pi$

$n = 6 + k \cdot 12.$

$0 < n < 50 \Rightarrow$

$n = 6 \vee n = 18 \vee n = 30 \vee n = 42.$



G36c $f_4(x) = 1 + \sin^2(x) + \cos(4x)$

$= 1 + \frac{1}{2} - \frac{1}{2}\cos(2x) + \cos(4x)$

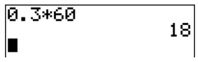
$= 1\frac{1}{2} - \frac{1}{2}\cos(2x) + \cos(4x).$

G36d
$$O(V) = \int_0^{2\pi} (4 - f_4(x))\,dx$$
$$= \int_0^{2\pi} \left(4 - \left(1\tfrac{1}{2} - \tfrac{1}{2}\cos(2x) + \cos(4x)\right)\right) dx$$
$$= \int_0^{2\pi} \left(2\tfrac{1}{2} + \tfrac{1}{2}\cos(\boxed{2x}) - \cos(\boxed{4x})\right) dx$$
$$= \left[2\tfrac{1}{2}x + \tfrac{1}{4}\sin(2x) - \tfrac{1}{4}\sin(4x)\right]_0^{2\pi}$$
$$= 5\pi + \tfrac{1}{4}\sin(4\pi) - \tfrac{1}{4}\sin(8\pi) - \left(0 + \tfrac{1}{4}\sin(0) - \tfrac{1}{4}\sin(0)\right)$$
$$= 5\pi + 0 - 0 - (0 + 0 - 0) = 5\pi.$$

O(rechthoek $ABCD$) $= 2\pi \cdot 5 = 10\pi.$

Dus de grafiek van f_4 verdeelt de rechthoek in twee gebieden met dezelfde oppervlakte.

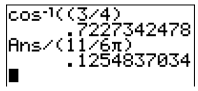
G 37a $x = 3\sin(2\pi t)$ en $y = 3\cos(2\pi t)$ geeft de ciirkel met middelpunt (0, 0) en straal 3.
$x = 2\sin(\frac{1}{6}\pi t)$ en $y = 2\cos(\frac{1}{6}\pi t)$ geeft de ciirkel met middelpunt (0, 0) en straal 2.
$t = 1{,}3$ de grote wijzer heeft $1\frac{3}{10}$ rondgang gemaakt $\Rightarrow$ het is 18 minuten over 1.



G37b Wijzers (niet de eindpunten) vallen over elkaar $\Rightarrow \sin(2\pi t) = \sin(\frac{1}{6}\pi t)$ en $\cos(2\pi t) = \cos(\frac{1}{6}\pi t)$.
$\left(2\pi t = \frac{1}{6}\pi t + k\cdot 2\pi \vee 2\pi t = \pi - \frac{1}{6}\pi t + k\cdot 2\pi\right)$ en tevens $\left(2\pi t = \frac{1}{6}\pi t + k\cdot 2\pi \vee 2\pi t = -\frac{1}{6}\pi t + k\cdot 2\pi\right)$
$\left(\frac{11}{6}\pi t = k\cdot 2\pi \vee \frac{13}{6}\pi t = \pi + k\cdot 2\pi\right)$ en tevens $\left(\frac{11}{6}\pi t = k\cdot 2\pi \vee \frac{13}{6}\pi t = k\cdot 2\pi\right)$
$\left(\frac{11}{6}t = k\cdot 2 \vee \frac{13}{6}t = 1 + k\cdot 2\right)$ en tevens $\left(\frac{11}{6}t = k\cdot 2 \vee \frac{13}{6}t = k\cdot 2\right)$
$\left(t = k\cdot\frac{12}{11} \vee t = \frac{6}{13} + k\cdot\frac{12}{13}\right)$ en tevens $\left(t = k\cdot\frac{12}{11} \vee t = k\cdot\frac{12}{13}\right)$
$t = k\cdot\frac{12}{11} \Rightarrow$ het eerste tijdstip na $t = 0$ is dus $t = \frac{12}{11}$.

G37c afstand $= \sqrt{\left(3\sin(2\pi t) - 2\sin(\frac{1}{6}\pi t)\right)^2 + \left(3\cos(2\pi t) - 2\cos(\frac{1}{6}\pi t)\right)^2}$
$= \sqrt{9\sin^2(2\pi t) - 12\sin(2\pi t)\sin(\frac{1}{6}\pi t) + 4\sin^2(\frac{1}{6}\pi t) + 9\cos^2(2\pi t) - 12\cos(2\pi t)\cos(\frac{1}{6}\pi t) + 4\cos^2(\frac{1}{6}\pi t)}$
$= \sqrt{9\left(\sin^2(2\pi t) + \cos^2(2\pi t)\right) + 4\left(\sin^2(\frac{1}{6}\pi t) + \cos^2(\frac{1}{6}\pi t)\right) - 12\left(\cos(2\pi t)\cos(\frac{1}{6}\pi t) + 12\sin(2\pi t)\sin(\frac{1}{6}\pi t)\right)}$
$= \sqrt{9\cdot 1 + 4\cdot 1 - 12\cos(2\pi t - \frac{1}{6}\pi t)} = \sqrt{13 - 12\cos(\frac{11}{6}\pi t)}$.

G37d Een gelijkbenige driehoek als
afstand $= 3 \quad \vee \quad$ afstand $= 2$
$\sqrt{13 - 12\cos(\frac{11}{6}\pi t)} = 3 \quad \vee \quad \sqrt{13 - 12\cos(\frac{11}{6}\pi t)} = 2$
$13 - 12\cos(\frac{11}{6}\pi t) = 9 \quad \vee \quad 13 - 12\cos(\frac{11}{6}\pi t) = 4$
$-12\cos(\frac{11}{6}\pi t) = -4 \quad \vee \quad -12\cos(\frac{11}{6}\pi t) = -9$
$\cos(\frac{11}{6}\pi t) = \frac{1}{3} \quad \vee \quad \cos(\frac{11}{6}\pi t) = \frac{3}{4}$
Het eerste moment na $t = 0$ ($\cos(0) = 1$) volgt uit $\cos(\frac{11}{6}\pi t) = \frac{3}{4} \Rightarrow \frac{11}{6}\pi t \approx 0{,}723 \Rightarrow t \approx 0{,}125$.



G38a $f(x) = \sin(x) \Rightarrow T = (\frac{1}{2}\pi, 1)$ en $A(\pi, 0)$.
$g(0) = \frac{-4}{\pi^2}\cdot 0\cdot(0 - \pi) = 0 \Rightarrow$ de grafiek van g gaat door O.
$g(\frac{1}{2}\pi) = \frac{-4}{\pi^2}\cdot\frac{1}{2}\pi\cdot(\frac{1}{2}\pi - \pi) = \frac{-4}{\pi^2}\cdot\frac{1}{2}\pi\cdot -\frac{1}{2}\pi = \frac{-4}{\pi^2}\cdot -\frac{1}{4}\pi^2 = 1 \Rightarrow$ de grafiek van g gaat door T.
$g(\pi) = \frac{-4}{\pi^2}\cdot\pi\cdot(\pi - \pi) = \frac{-4}{\pi^2}\cdot\pi\cdot 0 = 0 \Rightarrow$ de grafiek van g gaat door A.

G38b $f(x) = \sin(x) \Rightarrow f'(x) = \cos(x)$.
$g(x) = -\frac{4}{\pi^2}x\cdot(x - \pi) = -\frac{4}{\pi^2}x^2 + \frac{4}{\pi}x \Rightarrow g'(x) = -\frac{8}{\pi^2}x + \frac{4}{\pi}$.
$f'(0) = \cos(0) = 1$ en $g'(0) = -\frac{8}{\pi^2}\cdot 0 + \frac{4}{\pi} = \frac{4}{\pi} > 1 \Rightarrow g'(0) > f'(0)$.

G38c $\int_0^\pi (g(x) - f(x))\,dx = \int_0^\pi (ax(x - \pi) - \sin(x))\,dx \int_0^\pi \left(ax^2 - a\pi x - \sin(x)\right)dx = \left[\frac{1}{3}ax^3 - \frac{1}{2}a\pi x^2 + \cos(x)\right]_0^\pi$
$= \frac{1}{3}a\pi^3 - \frac{1}{2}a\pi\cdot\pi^2 + \cos(\pi) - \left(\frac{1}{3}a\cdot 0^3 - \frac{1}{2}a\pi\cdot 0^2 + \cos(0)\right) = \frac{1}{3}a\pi^3 - \frac{1}{2}a\pi^3 - 1 - (0 - 0 + 1) = -\frac{1}{6}a\pi^3 - 2$.
$\int_0^\pi (g(x) - f(x))\,dx = 0 \Rightarrow -\frac{1}{6}a\pi^3 = 2 \Rightarrow a\pi^3 = -12 \Rightarrow a = -\frac{12}{\pi^3}$.

$\sin(-A) = -\sin(A)$ $\cos(-A) = \cos(A)$
$-\sin(A) = \sin(A+\pi)$ $-\cos(A) = \cos(A+\pi)$
$\sin(A) = \cos(A-\frac{1}{2}\pi)$ $\cos(A) = \sin(A+\frac{1}{2}\pi)$
$\sin^2(A)+\cos^2(A) = 1$ $\tan(A) = \frac{\sin(A)}{\cos(A)}$

$$\sin(2A) = 2\sin(A)\cos(A)$$
$$\cos(2A) = \cos^2(A)-\sin^2(A)$$
$$= 2\cos^2(A)-1$$
$$= 1-2\sin^2(A)$$

$$\cos(t+u) = \cos(t)\cdot\cos(u)-\sin(t)\cdot\sin(u)$$
$$\cos(t-u) = \cos(t)\cdot\cos(u)+\sin(t)\cdot\sin(u)$$
$$\sin(t+u) = \sin(t)\cdot\cos(u)+\cos(t)\cdot\sin(u)$$
$$\sin(t-u) = \sin(t)\cdot\cos(u)-\cos(t)\cdot\sin(u)$$

$$\sin(A) = \sin(B) \Rightarrow A = B+k\cdot 2\pi \vee A = \pi-B+k\cdot 2\pi$$
$$\cos(A) = \cos(B) \Rightarrow A = B+k\cdot 2\pi \vee A = -B+k\cdot 2\pi$$

De grafiek van de functie f is symmetrisch in de lijn $x = a$ als voor elke p geldt: $f(a-p) = f(a+p)$.

De grafiek van de functie f is symmetrisch in het punt (a, b) als voor elke p geldt: $\frac{f(a-p)+f(a+p)}{2} = b$.

Dus de grafiek van de functie f is symmetrisch in het punt (a, b) als voor elke p geldt: $f(a-p)+f(a+p) = 2b$.

$f(x)$	afgeleide $f'(x)$
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$\tan(x)$	$\frac{1}{\cos^2(x)}$ of $1+\tan^2(x)$
$f(ax+b)$	$a\cdot f'(ax+b)$

$f(x)$	primitieven $F(x)$
$\sin(x)$	$-\cos(x)+c$
$\cos(x)$	$\sin(x)+c$
$1+\tan^2(x)$ of $\frac{1}{\cos^2(x)}$	$\tan(x)+c$
$f(ax+b)$	$\frac{1}{a}\cdot F(ax+b)+c$